$(t_1, h_1, h_2, h_3) \cdot (t_2, g_1, g_2, g_3) =$
$$= (t_1 + t_2, h_1 + g_1, e^{h1}(g_2\cos t_1 - g_3\sin t_1) + h_2, e^{h1}(g_2\sin t_1 + g_3\cos t_1) + h_3).$$
The inverse element:
$$(t, h_1, h_2, h_3)^{-1} = (-t, -h_1, e^{-h1}(h_2\cos at + h_3\sin at), e^{-h1}(-h_2\sin at + h_3\cos at)).$$
Transformations of the second kind can also be represented as a matrix, but the product of two transformations of the second kind is a transformation of the first kind.

Notice, that
$$(t_1, h_1, 0, 0) \cdot (t_2, g_1, 0, 0) = (t_1 + t_2, h_1 + g_1, 0, 0).$$
$$(0, 0, h_2, h_3) \cdot (0, 0, g_2, g_3) = (0, 0, h_2 + g_2, h_3 + g_3).$$
This means that elements for which $t = h_1 = 0$ form a commutative subgroup, and elements for which $h_2 = h_3 = 0$ also form a commutative subgroup. Let us denote these subgroups $G_2$ and $G_1$ respectively, and assign two coordinates to their elements, discarding zeros. Then the unit element in each group has coordinates (0, 0), the inverse elements are:
$$(t, h_1)^{-1} = (-t, -h_1),\ (h_2, h_3)^{-1} = (-h_2, -h_3).$$
Thus, both groups $G_1$ and $G_2$ are isomorphic to $\boldsymbol{R}^2$ with the addition operation.

We assign each element $(t_1, h_1) \in G_1$ a transformation $\varphi(t, h_1) : G_2 \to G_2$, which acts by formula:
$$\varphi(t, h_1)(h_2, h_3) = (e^{h1}(h_2\cos t_1 - h_3\sin t_1), e^{h1}(h_2\sin t_1 + h_3\cos t_1)).$$
It is easy to check that
$$\varphi(t_1 + t_2, h_1 + g_1)(h_2, h_3) = \big(\varphi(t_1, h_1) \circ \varphi(t_2, g_1)\big)(h_2, h_3) = \big(\varphi(t_2, h_2) \circ \varphi(t_1, h_1)\big)(h_2, h_3).$$
Since each of the transformations $\varphi(t, h_1)$ are linear, it is an automorphism of the group $G_2$. Thus, we have the mapping $\varphi : G_1 \to \mathrm{Aut}(G_2)$, which is a homomorphism.

From this, we can conclude that the following theorem is true.

**Theorem.** *Group $I_e(S_3)$ is a semidirect product of the groups $G_1$ and $G_2$: $I_e(S_3) = G_2 \rtimes_\varphi G_1$. Each of the groups $G_1$ and $G_2$ is isomorphic to the group $\boldsymbol{R}^2$ with the addition operation.*

**Conclusion.** In this paper we have supplemented the results published in [1]. We have found a matrix representation of the full isometry group of a homogeneous Riemannian manifold of a special three-dimensional Lie group. It turned out that the full isometry group is a semidirect product of two two-dimensional commutative Lie groups. Based on the previously obtained results, an article was published in a scientific journal from the list of the Higher Attestation Commission of the Russian Federation.


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## IMPLEMENTATION OF ALGORITHMA FOR DIVIDING THE ARC OF A CIRCLE INTO SECTIONSOF EQUAL LENGTH


***Wang Wenfeng,***
*1st year master's student, VSU named P.M. Masherov, Vitebsk, Republic of Belarus*
*Scientific supervisor – Buyevich A.E., candidate of engineering sciences, docent*



Keywords. Algorithm, coordinates, computer-aided design, control programme.


The article considers the development and implementation of an algorithm for obtaining the trajectory of the actuator movement along given coordinates on a circular arc. The software module allows you to expand the capabilities in enterprises of computer-aided design systems operating at enterprises in the development of control programs for technological equipmentю with electronic control in light industry.

Target of the research – development and realization of the algorithm of line division into elements for integrated CAD.

**Material and methods.** It is also possible to automate the solution of production tasks without using additional software by integrating modules that expand their capabilities into the enterprise's computer-aided design systems.

**Results and their discussion.** In mechatronic systems, it is often necessary to work out the movements of the actuator along given coordinates. An algorithm is developed for dividing the trajectory and in the form of a circular arc into nodes (points) at a given distance $n_0$ from each other. Fig. 1 shows a calculation scheme of the algorithm for dividing an arc into sections of equal length. In presenting the arc is represented in vector form by the coordinates of the end-points 1, 2 and the curvature coefficient r. Figure 1 shows: point 1 with coordinates $(x_1,y_1)$, point 2 with coordinates $(x_2 , y_2)$, $l_Д$ is length the arc length, $l$is the distance between points 1 and 2, $\Delta l$is the specified distance between neighboring points, R is the arc radius, F is the central angle.

The source data about the circle arc created in the AutoCAD system is contained in the drawing exchange file, which is a plain text file of the "*.dxf" type in ASCII codes. The "*.dxf file" contains text information about the vector image in a specially specified format. The coordinates of points 1 and 2 at the beginning and end of the circle arc (polyline) graphic primitive are contained in groups defined by the corresponding codes. For example, the group code "10" indicates the primary coordinate X (the starting point of a line or text graphic primitive, the center of a circle, etc.); the group code " 20 " « 20» indicates the primary coordinate Y. The values of the second coordinate always correspond to the values of the first coordinate and follow them directly in the file. The code " 42" means that the transition to the next point is made in an arc. The arc width r is written in the following line of the "*.dxf file" and is expressed by a coefficient that takes values from -1 to 1. A fragment of the program for determining the initial data on the circular arc is shown in Figure 2.

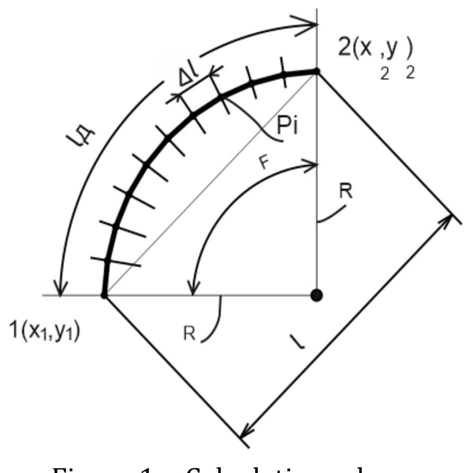


Figure 1 – Calculation scheme

```
If st=' 10' then
 Begin
    Readln(Var_f,st);
    Val(st,cr,cod);
     x2:=cr;
 end;
x1:= x2;

If st=' 20' then
 Begin
     Readln(Var_f,st);
     Val(st,cr,cod);
      y2:=cr;
 end;
y1:= y2;

If st=' 42' then
 Begin
     Readln (Var_f,st);
     Val (st,cr,cod);
     R:=Cr;
 end;
```

Figure 2 – Fragment of the program for determining the initial data about the arc

Given the known coordinates of point 1 ($x_1$ , $y_1$) and point 2 ($x_2$ , $y_2$) p, the distance between points 1 and 2 is determined by the expression:

$$l = \sqrt{((x2 - x1)^2 + (y2 - y1)^2)}. \quad (1)$$

The curvature coefficient r is used to calculate the radius R and the central angle F:

$$R = \frac{r*l + l/r}{4}; \quad (2)$$

$$F = arccos\frac{(2*R^2 - l^2)}{2*R}. \quad (3)$$

The value of the auxiliary angles $F_1$ is determined, which complement the central angle F up to 180 degrees on both sides.

An implementation of the algorithm for calculating the distance l between points 1 and 2, the radius a of the arc R, the central angle F, and the auxiliary angles $F_{f1}$ is shown in Figure 3.

```
l:=sqrt(sqr(x2-x1)+sqr(y2-y1));
R:=((R*l+l/R)/4);
F:=arccos((2*sqr(R)-sqr(l))/(2*sqr(R)));
F1:=(Pi-F)/2;...
```

Figure 3 – Fragment of the program for calculating l, R and F

The arc length is calculated and an integer N segments of length$_0$, are found, which are placed in the found length:

$$l_Д = F * R; \quad (4)$$

$$N = \left\lceil \frac{ld}{n0} \right\rceil. \quad (5)$$

Determined y-step $\Delta f$ and linear increments $\Delta x, \Delta y$ along the coordinate axes are determined:

$$\Delta f = \frac{F}{N}; \quad (6)$$

$$\Delta x = R * \cos(\Delta f); \quad (7)$$

$$\Delta y = R * \sin(\Delta f). \quad (8)$$

Figure 4 shows a fragment of the program for determining the number of N segments and specifying their length. The variable Ld describes the length of an arc of radius R, the variable Lim describes the approximate user-specified distance $n_0$, the variable Koef specifies the number of segments of length $n_0$ that fit in the arc length between points 1 and 2, the variables Fi, Xrel, Yrel describe, respectively, the angular step and linear increments in coordinate coordinates axes.

```
Ld:=F*R;
Koef:=Floor(Ld /Lim);
if Koef<=1 then begin Koef:=1 end;
Fi:=Fi/Koef;
Koef:=abs(Koef);
Xrel:=R*cos(F1);
Yrel:=R*sin(F1);
```

Figure 4 –Fragment of the program for determining the number and specifying the length of segments

After that, the coordinates of neighboring points to Pi(xi,yi) of the arc are determined:

$$x_{i=}x_{i-1}+\frac{\Delta x(x_2-x_1)}{l}-\frac{\Delta y(y_2-y_1)}{l}; \quad (9)$$

$$y_{i=}y_{i-1}+\frac{\Delta y(x_2-x_1)}{l}+\frac{\Delta x(y_2-y_1)}{l}. \quad (10)$$

Figure 5 shows a fragment of a program for calculating the coordinates of Pi points be longing to an arc. The variables Xr, Yr determine the coordinates (xi,yi) of the current arc point. The coordinate values of the current point are found incrementing along the arc of the coordinates of the starting point, denoted by the variables X1,Y1. After the calculation, the coordinate values of the current point are assigned to the new starting point and denoted by the times X2,Y2.The calculation cycle is repeated Koef times. Using the Write operator, the calculated coordinates of the points of the circle arc c by means of an operator Write are saved to a file that is passed to the integrated CAD system.

```
Xr:=X1+Xrel*((X2-X1)/l)-Yrel*((Y2-Y1)/l);
Yr:=Y1+Xrel*((Y2-Y1)/l)+Yrel*((X2-X1)/l);
for n:=1 to Koef do
  Begin
    X2:=Xr;
     Y2:=Yr;
     Write(Inp_f,'"');
     Write(Inp_f,X2_p:cod:10,',');
      Write(Inp_f,Y2_p:cod:10);
       WriteLn(Inp_f,'"');
  End;
```

Figure 5 – Fragment of the program for calculating the coordinates of pointsofan arc

**Conclusion.** The developed software module is designed to calculate the coordinates of points located on the arc of a circle with a given step, and determine the trajectory of the actuator movement.

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## REALISATION OF THE ALGORITHM OF DIVISION OF A LINE SEGMENT INTO EQUAL SEGMENTS

***Wang Wenfeng,***
*master's student, VSU named P.M. Masherov, Vitebsk, Republic of Belarus*
*Scientific supervisor – Buyevich A.E., candidate of engineering sciences, docent*

Keywords. Algorithm, coordinates, computer-aided design, control programme.

The paper is devoted to the development and implementation of an algorithm for dividing a straight line segment into equal sections. The proposed software module can be used for obtaining trajectories of movement of the actuator according to the given coordinates on laser complexes, cutting machines, sewing semiautomatic machines in light industry.

Target of the research – development and realization of the algorithm of line division into elements for integrated CAD.

**Material and methods.** The work is based on the results of analysis of scientific and technical information on computer-aided design systems, integrated systems, automated equipment; experimental work on the study of technological processes of parts processing on automated equipment; use of computer modeling methods.