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COMPLETE ISOMETRY GROUP OF A HOMOGENEOUS MANIFOLD OF A SPECIAL NON-UNIMODULAR LIE GROUP

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In paper [1] we found the matrix representation and full group of isometries the homogeneous manifold of the special three-dimensional non-unimodular Lie group S_3 equipped with left-invariant Riemannian metric. The purpose of this paper is to find a matrix representation of this group of isometries and to study the structure of this group.

Material and methods. We consider the special non-unimodular three-dimensional Lie group S_3 , equipped with left-invariant Rimannian metrics, and the full group of isometries of the resulting homogeneous manifold. We use methods of algebra and differential geometry.

Results and its discussion. In paper [1] we found the matrix representations of Lie group S_3 and its Lie algebra $\mathcal{S}_3$. These representations helped us to introduce the natural coordinates in S_3 and $\mathcal{S}_3$. Then we found the canonical form of the left-invariant metric tensor in the natural coordinates, and we found the full group of isometries of the resulting homogeneous manifold. This group is four-dimensional, not connected, and with respect to the natural coordinates it acts according to the formulas

$$\begin{cases} x_1' = x_1 + h_1, \\ x_2' = (x_2\cos at - x_3\sin at)e^{h1} + h_2, \\ x_3' = (x_2\sin at + x_3\cos at)e^{h1} + h_3, \end{cases} \tag{1}$$

$$\begin{cases} x_1' = x_1 + h_1, \\ x_2' = (x_2\sin at + x_3\cos at)e^{h1} + h_2, \\ x_3' = (x_2\cos at - x_3\sin at)e^{h1} + h_3, \end{cases} \tag{2}$$

where $H(h_1, h_2, h_3)$ is an arbitrary element of the Lie group S_3, $t\in\boldsymbol{R}$, $a \neq 0$.

We will call the transformations, which act by these formulas the isometries of the first and of the second kind respectively. Only the transformations of the first kind form the group, and we denote this group as $I_e(S_3)$.

If we consider the implementation of the Lie group S_3 in the form of column vectors with components $(x_1, 1, x_2, x_3, 1)$, then the action of transformations from $I_e(S_3)$ can be represented as a matrix

$$[T] = \begin{pmatrix} 1 & h_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & e^{h1}\cos at & -e^{h1}\sin at & h_2 \\ 0 & 0 & e^{h1}\sin at & e^{h1}\cos at & h_3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Let us assign coordinates (t, h_1, h_2, h_3) to such transformation. The coordinate t is cyclic: coordinates (t, h_1, h_2, h_3) and $\left(t + \frac{2\pi}{a}, h_1, h_2, h_3\right)$ define the same point. Suppose, that we have two transformations with coordinates (t_1, h_1, h_2, h_3) and (t_2, g_1, g_2, g_3). Then, by multiplying the corresponding matrices, we find that the group operation in $I_e(S_3)$ is given by the formulas

$(t_1, h_1, h_2, h_3) \cdot (t_2, g_1, g_2, g_3) =$

$$= (t_1 + t_2, h_1 + g_1, e^{h1}(g_2 \cos t_1 - g_3 \sin t_1) + h_2, e^{h1}(g_2 \sin t_1 + g_3 \cos t_1) + h_3).$$

The inverse element:

$$(t, h_1, h_2, h_3)^{-1} = (-t, -h_1, e^{-h1}(h_2 \cos at + h_3 \sin at), e^{-h1}(-h_2 \sin at + h_3 \cos at)).$$

Transformations of the second kind can also be represented as a matrix, but the product of two transformations of the second kind is a transformation of the first kind.

Notice, that

$$(t_1, h_1, 0, 0) \cdot (t_2, g_1, 0, 0) = (t_1 + t_2, h_1 + g_1, 0, 0).$$
$$(0, 0, h_2, h_3) \cdot (0, 0, g_2, g_3) = (0, 0, h_2 + g_2, h_3 + g_3).$$

This means that elements for which $t = h_1 = 0$ form a commutative subgroup, and elements for which $h_2 = h_3 = 0$ also form a commutative subgroup. Let us denote these subgroups G_2 and G_1 respectively, and assign two coordinates to their elements, discarding zeros. Then the unit element in each group has coordinates (0, 0), the inverse elements are:

$$(t, h_1)^{-1} = (-t, -h_1), \ (h_2, h_3)^{-1} = (-h_2, -h_3).$$

Thus, both groups G_1 and G_2 are isomorphic to $\boldsymbol{R}^2$ with the addition operation.

We assign each element $(t_1, h_1) \in G_1$ a transformation $\varphi(t, h_1) : G_2 \to G_2$, which acts by formula:

$$\varphi(t, h_1)(h_2, h_3) = (e^{h1}(h_2 \cos t_1 - h_3 \sin t_1), e^{h1}(h_2 \sin t_1 + h_3 \cos t_1)).$$

It is easy to check that

$$\varphi(t_1 + t_2, h_1 + g_1)(h_2, h_3) = \big(\varphi(t_1, h_1) \circ \varphi(t_2, g_1)\big)(h_2, h_3) = \big(\varphi(t_2, h_2) \circ \varphi(t_1, h_1)\big)(h_2, h_3).$$

Since each of the transformations $\varphi(t, h_1)$ are linear, it is an automorphism of the group G_2. Thus, we have the mapping $\varphi : G_1 \to \mathrm{Aut}(G_2)$, which is a homomorphism.

From this, we can conclude that the following theorem is true.

Theorem. *Group $I_e(S_3)$ is a semidirect product of the groups G_1 and G_2: $I_e(S_3) = G_2 \rtimes_\varphi G_1$. Each of the groups G_1 and G_2 is isomorphic to the group $\boldsymbol{R}^2$ with the addition operation.*

Conclusion. In this paper we have supplemented the results published in [1]. We have found a matrix representation of the full isometry group of a homogeneous Riemannian manifold of a special three-dimensional Lie group. It turned out that the full isometry group is a semidirect product of two two-dimensional commutative Lie groups. Based on the previously obtained results, an article was published in a scientific journal from the list of the Higher Attestation Commission of the Russian Federation.

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IMPLEMENTATION OF ALGORITHMA FOR DIVIDING THE ARC OF A CIRCLE INTO SECTIONSOF EQUAL LENGTH

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Keywords. Algorithm, coordinates, computer-aided design, control programme.

The article considers the development and implementation of an algorithm for obtaining the trajectory of the actuator movement along given coordinates on a circular arc. The software module allows you to expand the capabilities in enterprises of computer-aided design systems