KREATIVITÄT IM NORMALEN MATHEMATIKUNTERRICHT

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Kreativ zu sein ist eines der wichtigsten aber auch mit am schwersten zu erreichenden Ziele für jeden Unterricht. Anliegen dieses Artikels ist es aufzuzeigen, dass dieses Ziel auch schon in ganz normalem Mathematikunterricht erreichbar ist. Hierfür sind entsprechend qualifizierte Lehrer erforderlich. Man kann versuchen, wie in Finnland diese durch entsprechende Auswahlverfahren zu gewinnen.

Material und Methoden. Zunächst wird versucht, den Begriff „Kreativität" unter Heranziehung vor allem psychologischer Literatur zu klären. Hermeneutische Analysen der Geschichte der Mathematik haben acht kreative Tätigkeiten ergeben, die im Laufe von über 5000 Jahren immer wieder neue mathematische Erkenntnisse hervorgebracht haben. Fallstudien aus der Bruchrechnung sollen verdeutlichen helfen, wie auf solchem Hintergrund die Förderung mathematischer Kreativität im ganz normalen Unterricht möglich sein kann.

Ergebnisse und Diskussion. Historische Analysen ergaben ein „Oktagon" von acht miteinander zusammenhängenden produktiven Tätigkeiten, die auch für den heutigen Unterricht nützlich sind.

Fallstudien machen deutlich, dass nicht nur begabte Schüler unter entsprechender Anleitung mathematisch kreativ sein können.

Auswahlverfahren können helfen, kreative Lehramtskandidaten zu bestimmen.

Schlussfolgerung. Kreative Lehrer sind unverzichtbar für einen kreativen Unterricht.

Schlüsselworte. Kreativität, Mathematikunterricht, Geschichte mathematischer Kreativität, Test für kreative Lehrer

КРЕАТИВНОСТЬ В ПОВСЕДНЕВНЫХ УРОКАХ МАТЕМАТИКИ

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Креативный подход является одной из важнейшейших, а одновременно и трудно достижимых целей стоящих перед преподавателями средних школ. В настоящей статье будет показано, что названная цель может быть достигнута уже в рамках уроков математики. Необходимым условием для этого является наличие соответственной квалификации у преподавательского состава. Это свойство может быть выработанно - как, к примеру, в Финляндии - путём подбора будущих учителей,то есть студентов.

Материал и методика. В статье будет предпринята попытка, операясь на научные публикации из области психологии, дать определение понятию "креативность". В результате анализа истории математики было установлено восемь креативных методов, которые на протяжении более чем пяти тысяч лет неоднократно сбособствоали совершению новых открытий в области математики. Исследования дробей служат пояснением тому, как можно способствовать развитию креативновсти в повседневных уроках математики.

Обсуждение результатов. В результате анализа с исторической перспективы на сегодняшний день известны восемь взаимосвязанных областей деятельности, которые могут быть применены в современных уроках математики. Исследования показывают, что при соответственном преподавании не только способные ученики открывают креативный подход к математике.

Выводы. Методика подбора студентов в вышие педагогические институты позволяет выявить креативные кадры для школы.

Ключевые слова. Креативность, преподавание математики, история креативности в математике, подбор креативных преподавателей.

After Hilbert was told that a student in his class had dropped mathematics in order to become a poet, he is reported to have said "Good - he did not have enough imagination to become a mathematician"

Quoted from Hoffman 1998, p. 95.

Introduction. As I have been a mathematics school-teacher and a mathematics educator at university for many years, I will concentrate on mathematical creativity of pupils and teachers in mathematics instruction.

The quotation above should help to highlight that creativity in mathematics can be seen as not less important than in the art of poetry. David Hilbert (1862 - 1943) was one of the most famous mathematicians of the last century. One might be surprised by this statement, because mathematics seems to be a fixed body of eternal truth where it is not to possible to discover (or to invent) something new.

This impression is delusive. Such opinion might be generated rather often by own experience from school, in which a specific form of teaching of formulae, rules concepts and procedures is dominating.

What is creativity? Many useful hints can be found in the psychological literature which could help to answer this question (cf. list of references). Kaufman/Sternberg 2010, p. 467, present the following useful characterisation:

Typical for creative work is **originality** and **quality**.

If there was a creative act, some (relative) new and original object (concrete or abstract) should have been created. It should be new, at least, relative to the previous knowledge of the creative person (e. g., a pupil or student). A creative scientist, of course, should have invented or discovered some absolute new object, which nobody has invented or discovered before.

Writing a paper by always changing characters' colour from red to green consecutively may be original, but is obviously not very useful or relevant. Thus, the creative product should have also sufficient quality! But, what "quality" means is always determined by the judgment of distinguished experts in the respective domain.

Furthermore, creative behavior is related very often to specific properties of the personality as persistence, insistence, interest and curiosity.

Thus, creativity is related to a least three aspects: a specific domain (e. g. mathematics), personality and a specific social environment, which constitutes the judgment "creative".

How mathematics has been (and still is) invented? The conceptions about mathematics changed a lot in the course of its history. E. g., in ancient Greek (some 2000 years ago) „mathesis" was the subject-matter to be learned and someone was called „mathematikos" who was very ambitious to learn something. This ways of understanding illustrate the origin of today's denominations as well as the strong reduction of the original meanings of these concepts. One has to take this into account when studying the history of creativity in mathematics.

A long-term study of history of mathematics revealed eight main motives and activities, which proved to lead frequently to new mathematical results at different times and in different cultures for more than 5000 years (Fig. 1, cf. Zimmermann 2003):

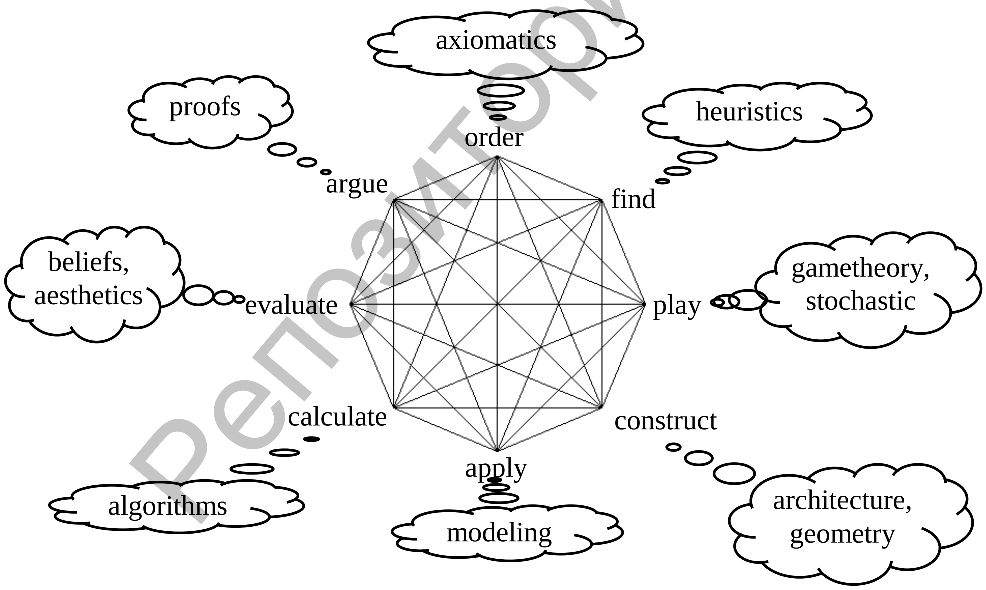


Fig. 1. **Eight sustainable activities which created again and again new mathematical content**

Some explanation is given to Fig. 1. There are three major groups of activities.

Calculating is at the beginning of nearly all mathematical actions. Problems, e. g., from astronomy and agriculture are until our days (cf. space industry and ecology) very important domains to ***apply*** mathematics or to develop new mathematical models, respectively. ***Constructing*** is the most important activity, not only in geometry but also in architecture – the latter one has been taken as a part of mathematics for a long time. These three activities are at the beginning of nearly all mathematical creations. We come now to a group of more sophisticated and challenging activities.

Arguing, esp. proving is at the core of modern mathematics and belongs to the more challenging mathematical actions. Of course, this activity is also related to ***finding*** methods (heuristics in the sense of the well known mathematician Pólya), which lead to conjectures first. Without inventions there are no proofs! The tension to bring new knowledge, a set of new theorems or clusters of solved problems in a systematic ***order***, might lead in the upper grades to first approaches to axiomatisation. This might help older and more mature students – practiced in appropriate situations and at appropriate time – to get a deeper understanding and more insight into theoretical connections.

The following two activities seem to be neglected rather often until now but proved to be of major importance for mathematical inventions, too. The striving for religious cognition and related systems of ***values*** generated frequently new problems and their solutions and produced in this way also new mathematical knowledge during history of mathematics. Systems of values are often related also to aesthetics, which may be sometimes still driving forces for mathematical inventions.

The same holds for an approach to mathematics by ***playing*** and the development of recreational mathematics. New branches of mathematics were a good many times created in this way like stochastic and game-theory.

These different activities - which are important, not only in mathematics - are connected and interrelated in many ways, which are represented in the figure by "diagonals".

One can take this network of activities as an element of a framework for the structuring of learning environments (e. g., for a textbook, cf. Cukrowicz/Zimmermann 2000-2006) and for analyzing students' cognitive and affective variables (cf. e. g. Eronen/Haapasalo 2010).

Creativity in the classroom. Many children normally start their school career full of curiosity, fantasy and creative potential. But many people complain of the fact, that these properties unfortunately very often are expelled - because of too many rigid constraints (cf. e. g. Bohm/Nichol 1998).

I want to demonstrate by examples from real classroom teaching (classical content "fractions"), that there are possibilities to preserve and foster these good properties.

Example 1: Comparing fractions

We are in a class of sixth-graders in a Hamburg comprehensive school of a workers-district with some 80% of migrants. Many of them are not capable to speak German fluently - so, obviously, it was no elitist situation. In the previous lessons fractions were treated, the pupils had learned already how to expand and reduce fractions.

The following lesson was the first one about ordering of fractions.

The teacher wrote at the blackboard and asked the pupils:

Problem 1: How would you arrange the following fractions according to their size: $\frac{5}{6}, \frac{3}{7}, \frac{2}{3}$?

We invite the reader to stop reading here and solving this little problem her- or himself.

Now please think a little bit about your strategies you applied!

Perhaps you remember the following strategy, which can be found often in schoolbooks:

"Compare fractions in the following way:

1. Determine their common denominator.

2. Multiply the respective nominator of each fraction by the same factor by which you have to multiply its denominator to get the common denominator.

3. Order now the size of the new nominators.

4. The order of these fractions yields the order of the corresponding equivalent original fractions."

Your own experience and that one of several teachers may be often *quite different* from the following procedure, which was suggested by pupils of our class:

The fraction $\frac{3}{7}$ is smaller than $\frac{1}{2}$, the fraction $\frac{2}{3}$ is larger than $\frac{1}{2}$, therefore $\frac{3}{7} < \frac{2}{3}$.

Furthermore, $\frac{5}{6}$ is closer to 1 than $\frac{2}{3}$, therefore $\frac{2}{3} < \frac{5}{6}$.

Therefore, the final order is $\frac{3}{7} < \frac{2}{3} < \frac{5}{6}$ (without referring to a common denominator).

Example 2: Dividing fractions

We are still in the same class some time later. The pupils just learned the rule how to multiply fractions:

Two fractions are to be multiplied by multiplying the corresponding nominators and denominators.

The teacher asked now the pupils:

Problem 2: Could you conjecture a rule how to divide two fractions?

A pupil answered:

“To divide two fractions, I have to divide the nominator of the first fraction by the nominator of the second fraction and the denominator of the first fraction by the denominator of the second fraction.”

How would you react if you would be the teacher?

Stop reading for a while and try to think about some possible reactions.

The teacher never heard about a rule like this one! Of course he intended to come to the well known rule:

“You have to divide two fractions by multiplying the first one by the reciprocal of the second one.”

I (the author of this contribution) presented this scene to my student teachers and asked them: How would you react if you would have been the teacher of this class?

Answers came as “I would try to help the pupils to understand, why this conjecture is wrong.”

The teacher of this class wasn’t sure about this conjecture either, but he was a very experienced and sensitive teacher with sufficient self-confidence. So he said: „Let’s check this conjecture by simple examples”.

So let us start with $\frac{4}{9} : \frac{2}{3}$.

The pupils calculated $\frac{4:2}{9:3} = \frac{2}{3}$. Of course one has to check the result by reversing the process: as $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$, therefore the equation $\frac{4}{9} : \frac{2}{3} = \frac{2}{3}$ is true. So the conjecture is true, if the nominator and denominator of the second fraction divide the nominator and the denominator of the first fraction, respectively.

What can we do, if the situation is not that easy? Let us continue carefully! E. g., what happens in case of $\frac{5}{9}:\frac{2}{3}$? When the door squeaks, you have to lubricate the hinges, to make them work smoothly again!

As 5:2 „squeaks“, perhaps we can make the division by 2/3 work by „lubricating“ the fraction $\frac{5}{9}$ by appropriate expansion (by 2): $\frac{5\cdot 2}{9\cdot 2}:\frac{2}{3}\overset{?}{=}\frac{(5\cdot 2):2}{(9\cdot 2):3}=\frac{5}{6}$. This equation holds, as the test yields $\frac{5}{6}\cdot\frac{2}{3}=\frac{5}{9}$. Therefore the conjecture is true also in this case.

What to do now with $\frac{5}{11}:\frac{2}{3}$?? We apply again the expanding-strategy, as it proved already to be successful in the previous case:

$$\frac{5}{11}:\frac{2}{3}=\frac{((5\cdot 2)\cdot 3)}{((11\cdot 2)\cdot 3)}:\frac{2}{3}=\frac{((5\cdot 2)\cdot 3):2}{((11\cdot 2)\cdot 3):3}\overset{?}{=}\frac{5\cdot 3}{11\cdot 2}=\frac{5}{11}\cdot\frac{3}{2}. \quad (*)$$

The first term equating the last term represents also the well known rule: To divide two fractions, the first fraction is to be multiplied with the reciprocal of the second. The rule is correct as it is demonstrated by the last check: If we multiply the last term in (*) by $\frac{2}{3}$, we get $\frac{5}{11}\cdot\frac{3}{2}\cdot\frac{2}{3}=\frac{5}{3}$. And indeed: The conjecture is always true, as the whole calculation in (*) is completely independent from the type of natural numbers we use in nominators and denominators. So it would not make a structural difference if we would use letters (names for variables) instead of concrete numbers. This would not change the mathematical outcome, but probably the learning outcome (esp. the understanding!) of the pupils!

Résumé:

- Pupils (and not only the gifted ones!) are able - even under difficult social conditions - to be creative in normal classroom instruction. They can reinvent standard-rules themselves - given appropriate guidance by a sensitive teacher.
- To increase the possibility for developments like the aforementioned ones, a talented and well educated teacher is essential, who is sensitive towards the pupils as well as towards the subject. Such teacher has to be sufficient curious, courageous and creative him- or herself.
- To put it together: Pupils are in need of creative teachers to become or to stay creative themselves.

Creative learning needs creative teachers. The foregoing examples may help to highlight the following consequence: The prevalent experience of a monotone, routine-oriented learning, we mentioned at the beginning, can only be reduced or moved in another direction by a different way of teaching. There is no lack of good suggestions and advice for formation of creativity-oriented instruction (cf. e. g. Kießwetter 1977).

The crucial point is: You need teachers who are able to implement such advice!

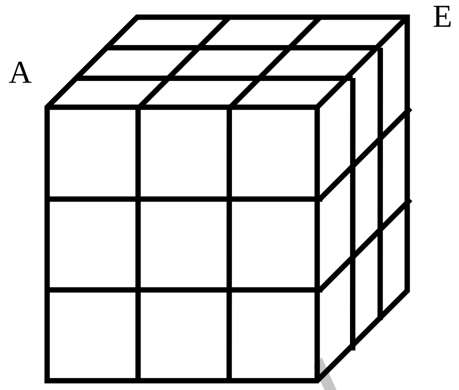
To reach this goal, there is for example the possibility, to use appropriate methods (tests) to select the right student teacher candidates, as it is practiced in Finland already for a long time. During the last years, only some 5 % out of all applicants are admitted for elementary teacher studies (grade 1 - 6).

The selection procedures are different depending on the university. In addition to educational conceptions and ability, cooperation skills in group discussions and reading comprehension of foreign-language scientific reading, the subject-matter related creativity is tested.

We present an example by which such creativity is tested:

Given the following cube with a uniform square-grid on all its faces:

Create as many as possible reasonable mathematical questions fitting to this situation.



Some possibilities:

- Let us imagine that the cube is made of wax. The cube is cut along the grid lines in such a way that 3x3x3 small single cubes are created. After each cut you may rearrange the resulting cuboids. Is it possible to produce these 27 little cubes by less than 6 cuts?
- Now the cube is painted red. How many small cubes are there with 3, 2, 1 or no red faces?
- One out of these 27 small cubes is removed from one corner / edge of the large cube. How does its volume and its surface change, respectively?
- What happens to the volume and the surface of the remaining solid when I remove ... 2, 3 to 26 small cubes?
- A beetle sits in the corner A of the cube and will migrate to the corner E. If the edge of a small cube has the length 1, how long is the shortest path from A to E? Are there several ways?
- Is it possible to distribute numbers 1-6 on the faces of the large cube so that - when rolling the dice - the probabilities for all numbers are different and not always 1/6?

  What are the options if applicable and how do they look like?
- How could a three-dimensional tic-tac-toe game look like?
- Do you find more reasonable possibilities?

References

1. Bohm, D.; Nichol, L. (eds. 1998). *On Creativity*. London, New York: Routledge.
2. Cukrowicz, J., Zimmermann, B. (eds. and coauthors, 2000-2006). *MatheNetz Ausgabe N; Gymnasien*. Braunschweig: Westermann Schulbuchverlag.
3. Glatfeld, M. (ed. 1977). *Mathematik Lernen. Probleme und Möglichkeiten.* Braunschweig: Vieweg.
4. Eronen, L., Haapasalo, L. (2010). Making Mathematics through Progressive Technology. In: Sriraman Bharath, Bergsten Christer, Goodchild Simon, Palsdottir Gudbjörn, Dahl Bettina, Haapasalo Lenni (eds. 2010), *The First Sourcebook on Nordic Research in Mathematics Education.* Information Age Publishing. The Montana Mathematics Enthusiast: Monograph Series in Mathematics Education. 701-710.
5. Haapasalo, L.; Sormunen, K. (eds. 2003). *Towards Meaningful Mathematics and Science Education*. Proceedings on the 19th Symposium of the Finnish Mathematics and Science Education Research Association. University of Joensuu. Bulletins of the Faculty of Education 86.
6. Hadamard, J. (1996). *The Mathematician's Mind. The Psychology of Invention in the Mathematical Field.* With a new preface by P. N. Johnson-Laird. Princeton, New Jersey: Princeton University Press.
7. Hoffman, P. (1998). *The Man Who Loved Only Numbers.* New York: Hyperion.
8. Kaufman, J. C.; Sternberg, R. J. (eds. 2010). *The Cambridge Handbook of Creativity.* New York: Cambridge University Press.
9. Kießwetter, K. (1977). Kreativität in der Mathematik und im Mathematikunterricht. In: Glatfeld 1977, 1-39.
10. Sternberg, R. J. (1988). *The Nature of Creativity*. New York: Cambridge University Press.
11. Zimmermann, B. (2003). On the Genesis of Mathematics and Mathematical Thinking - a Network of Motives and Activities drawn from the History of Mathematics. In: Haapasalo/Sormunen 2003, 29 - 47.