Conclusion. For the Cauchy problem for the nonlinear pseudoparabolic equation, we show the stabilization of the solution to the solution of the Cauchy problem for an ordinary differential equation constructed according to the original equation.

1. Kavitova, T.V. The existence of the solution of the Cauchy problem for some pseudoparabolic equation / T.V. Kavitova // Bulletin of VSU. – 2011. – № 3 (63), P. 14–19. URL: https://rep.vsu.by/handle/123456789/4528 (date of application: 06.11.2022).

2. Kavitova, T.V. Behavior of the maximal solution of the Cauchy problem for some nonlinear pseudoparabolic equation as $|x|\to\infty$ [Electronic resource] / T.V. Kavitova // Electron. J. Differ. Equ. – 2012. – Vol. 2012, № 141. – Mode of access: https://ejde.math.txstate.edu/Volumes/2012/141/ kavitova.pdf. – Date of access: 20.08.2012.

AUTOSIMILARITIES OF THE FOUR-DIMENSIONAL LORENTZIAN LIE ALGEBRA E(2)⊕*R*

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Let the Euclidean or Lorentz scalar product be given in the Lie algebra *G*. A linear transformation $f\colon \mathcal{G}\to\mathcal{G}$ is called an autosimilarity, if it is both an automorphism of the Lie algebra and a similarity with respect to the given inner product. The purpose of this paper is to find such the Lorentz scalar product on the four-dimensional Lie algebra $\mathcal{E}(2)\oplus\mathcal{R}$, for which this Lie algebra admits a one-parameter autosimilarity group.

Material and methods. We consider the four-dimensional Lie algebra $\mathcal{E}(2)\oplus\mathcal{R}$ equipped with the Lorentz scalar product. The methods of linear algebra and the theory of Lie algebras are used.

Findings and their discussion. The problem of finding self-similar homogeneous manifolds of a Lie group *G* equipped with the Riemannian or Lorentzian metric g involves initially solving the problem of finding one-parameter autosimilarity groups of the corresponding Lie algebra [1]. If such one-parameter groups exist, then the Lie algebra is also called self-similar.

Let $\mathcal{E}(2)$ – be the Lie algebra of the group of motions of the Euclidean plane. It was proved in [2], that this Lie algebra does not admit autosimilarity for any way of specifying the Lorentz scalar product on it. Consider the four-dimensional Lie algebra $\mathcal{G}_4=\mathcal{E}(2)\oplus\mathcal{R}$. In a suitable basis (E_1, E_2, E_3, E_4), the commutation relations are given by the equalities:

$$[E_1, E_2] = E_3,\ [E_1, E_3] = -E_2,$$

and the rest of the brackets are equal to the zero vector. Such a basis will be called canonical. In this paper, we will show that this Lie algebra can be self-similar, if the Lorentz scalar product is appropriately specified in it.

The Lie algebra $\mathcal{G}_4$ is solvable. It contains the three-dimensional commutative ideal $\mathcal{H}=\langle E_2, E_3, E_4\rangle$, one-dimensional center $\boldsymbol{R}E_4$, and the derived Lie algebra is two-dimensional: $\mathcal{G}_4^{(2)}=\mathcal{L}=\langle E_2, E_3\rangle$. In [3] a complete group of automorphisms of the considered Lie algebra was found and it was proved that it cannot be self-similar for any way of specifying a Euclidean scalar product on it.

Theorem. *Let the Lorentz scalar product in the Lie algebra* $\mathcal{G}_4 = \mathcal{E}(2)\oplus\mathcal{R}$ *be given using the Gram matrix in the canonical basis:*

$$\Gamma = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \qquad (1)$$

Then the given Lie algebra admits a one-parameter similarity group whose action in the canonical basis is given by the matrix

$$F(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{\nu t}\cos t & -e^{\nu t}\sin t & 0 \\ 0 & e^{\nu t}\sin t & e^{\nu t}\cos t & 0 \\ 0 & 0 & 0 & e^{2\nu t} \end{pmatrix}, \ \nu > 0,\ t \in \mathbf{R}. \qquad (2)$$

The location of the basis vectors relative to the cone of isotropic vectors is shown in Figure 1.

Proof. According to [3], the transformations $f_t : \mathcal{G}_4 \to \mathcal{G}_4$, which are given by matrix (2) in the canonical basis, are indeed automorphisms of the Lie algebra.

We can verify by direct calculation that

$$F^T(t)\Gamma F(t) = e^{2\nu t}\Gamma.$$

Thus, the transformations f_t are similarities with respect to the Lorentz scalar product, which is given by the matrix (1). It is easy to verify that the transformations f_t form a one-parameter group.

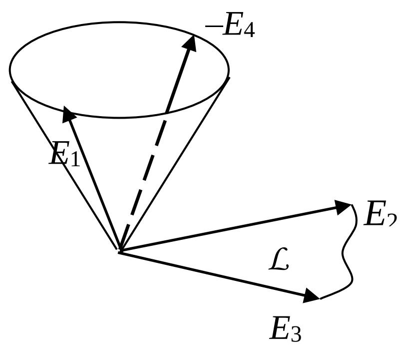

Figure 1

Conclusion. We proved that the Lie algebra $\mathcal{E}(2)\oplus\mathcal{R}$ equipped with the Lorentz scalar product can be self-similar and wrote out a matrix, that defines a one-parameter autosimilarity group in the canonical basis. The aim of the next research is to construct a self-similar homogeneous manifold of the Lie group $\mathcal{SE}(2)\times\mathcal{R}$, equipped with a left-invariant Lorentzian metric, and the found one-parameter autosimilarity group of the Lie algebra $\mathcal{E}(2)\oplus\mathcal{R}$ plays a decisive role in this. Moreover, we will try to construct a non-simply connected manifold, which belongs to the class of manifolds considered in [4].

1. Podoksyonov, M.N Similarities and isometries of the homogeneous manifold of the Heisenberg group with a left-invariant Lorentz metric // Bulletin of VSU. – 2011. – № 5. – P. 10–15. – Mode of access: https://rep.vsu.by/handle/123456789/4581. – Date of access: 30.10.2022).

2. Podoksyonov, M.N. Homothetic automorphisms of three-dimensional Lie algebras // Proceedings of P.M. Masherov University. Collection of scientific works. Vol. 8. Vitebsk, ed. of VSU, 2009. P. 203–211. – Mode of access: https://rep.vsu.by/handle/123456789/5125. – Date of access: 30.10.2022).

3. Podoksenov, M.N. Automorphisms and autometries of one four-dimensional Lie algebra of the Bianchi VI_2 subtype. / M.N. Podoksenov, Y.V. Gorovaya // Mathematical and computer modeling: a collection of materials of the IX International Scientific Conference dedicated to the 85th anniversary of Professor V.I. Potapov (Omsk, November 19, 2021). – Omsk: Publishing House of Omsk State University, 2021. – P. 24–26.

4. Podoksenov, M.N. A Lorentz manifold with a group of conformal transformations possessing a normal one-parameter homothety subgroup M.N. Podoksenov // Siberian mathematical journal, 1997. Vol. 38, № 6. – P. 1356–1359.