ON THE BEHAVIOR OF THE SOLUTION OF THE CAUCHY PROBLEM FOR A NONLINEAR PSEUDOPARABOLIC EQUATION AS $|x| \to \infty$

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In this work we describe the behavior of the solution of the Cauchy problem for a nonlinear pseudoparabolic equation for large values of a spatial variable.

Material and methods. Methods of the theory of partial differential equations.

Findings and their discussion. We consider the Cauchy problem for the nonlinear pseudoparabolic equation

$$\begin{cases} u_t = \Delta u_t + \Delta(u + u^2) - u^2, \quad x \in R^n, \ t > 0, \\ u(x,0) = u_0(x), \quad x \in R^n. \end{cases} \tag{1}$$

We suppose that the initial function $u_0(x)$ has the following properties:

$$u_0(x) \in C^2(R^n), \quad 0 \le u_0(x) \le M, \quad \lim_{|x|\to\infty} u_0(x) = M.$$

The existence and uniqueness of the solution $u(x,t)$ of problem (1) in the layer $R^n \times [0,T]$ (for any $T > 0$) is established in [1, 2].

We introduce an auxiliary problem for an ordinary differential equation

$$\begin{cases} v'(t) = -v^2(t), \quad t > 0, \\ v(0) = M. \end{cases} \tag{2}$$

Note, if $u_0(x) \equiv M, x \in R^n,$ then the solution of problem (2) is the solution of problem (1). According to the Picard's theorem problem (1) has unique solution, and $v(t)$ can be written in an explicit form

$$v(t) = \frac{M}{Mt+1}.$$

Put $u_0(x,t) = v(t)$. We define the sequence $u_n(x,t)$ $(n = 1,2,\ldots)$ as follows:

$$u_n(x,t) = u_0(x) - \int_0^t (u_{n-1}(x,\tau) + u_{n-1}^2(x,\tau))\, d\tau + \int_0^t \int_{R^n} \varepsilon(x-\xi) u_{n-1}(\xi,\tau)\, d\xi\, d\tau,$$

where $\varepsilon(x-\xi)$ is the fundamental solution of the operator $I - \Delta$ of R^n.

Lemma 1. The sequence $u_n(x,t)$ converges uniformly to $u(x,t)$ in the layer $R^n \times [0,T]$.

Lemma 2. For $n = 0,1,2,\ldots$ we have

$$u_n(x,t) \to v(t) \text{ as } |x| \to \infty$$

uniformly in $t \in [0,T]$.

Lemmas 1 and 2 imply the following statement.

Theorem. Let $u(x,t)$ be a solution of problem (1) and $v(t)$ be the solution of problem (2). Then we have

$$u(x,t) \to v(t) \text{ as } |x| \to \infty$$

uniformly in $t \in [0,T]$.

Conclusion. For the Cauchy problem for the nonlinear pseudoparabolic equation, we show the stabilization of the solution to the solution of the Cauchy problem for an ordinary differential equation constructed according to the original equation.

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AUTOSIMILARITIES OF THE FOUR-DIMENSIONAL LORENTZIAN LIE ALGEBRA E(2)⊕*R*

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Let the Euclidean or Lorentz scalar product be given in the Lie algebra *G*. A linear transformation $f\colon \mathcal{G}\to\mathcal{G}$ is called an autosimilarity, if it is both an automorphism of the Lie algebra and a similarity with respect to the given inner product. The purpose of this paper is to find such the Lorentz scalar product on the four-dimensional Lie algebra $\mathcal{E}(2)\oplus\mathcal{R}$, for which this Lie algebra admits a one-parameter autosimilarity group.

Material and methods. We consider the four-dimensional Lie algebra $\mathcal{E}(2)\oplus\mathcal{R}$ equipped with the Lorentz scalar product. The methods of linear algebra and the theory of Lie algebras are used.

Findings and their discussion. The problem of finding self-similar homogeneous manifolds of a Lie group *G* equipped with the Riemannian or Lorentzian metric g involves initially solving the problem of finding one-parameter autosimilarity groups of the corresponding Lie algebra [1]. If such one-parameter groups exist, then the Lie algebra is also called self-similar.

Let $\mathcal{E}(2)$ – be the Lie algebra of the group of motions of the Euclidean plane. It was proved in [2], that this Lie algebra does not admit autosimilarity for any way of specifying the Lorentz scalar product on it. Consider the four-dimensional Lie algebra $\mathcal{G}_4=\mathcal{E}(2)\oplus\mathcal{R}$. In a suitable basis (E_1, E_2, E_3, E_4), the commutation relations are given by the equalities:

$$[E_1, E_2] = E_3,\ [E_1, E_3] = -E_2,$$

and the rest of the brackets are equal to the zero vector. Such a basis will be called canonical. In this paper, we will show that this Lie algebra can be self-similar, if the Lorentz scalar product is appropriately specified in it.

The Lie algebra $\mathcal{G}_4$ is solvable. It contains the three-dimensional commutative ideal $\mathcal{H}=\langle E_2, E_3, E_4\rangle$, one-dimensional center $\boldsymbol{R}E_4$, and the derived Lie algebra is two-dimensional: $\mathcal{G}_4^{(2)}=\mathcal{L}=\langle E_2, E_3\rangle$. In [3] a complete group of automorphisms of the considered Lie algebra was found and it was proved that it cannot be self-similar for any way of specifying a Euclidean scalar product on it.