Mathematical Notes, vol. 78, no. 2, 2005, pp. 213–218.
Translated from Matematicheskie Zametki, vol. 78, no. 2, 2005, pp. 234–240.

Fitting Classes with Given Properties of Hall Subgroups

V. N. Zagurskii and N. T. Vorob′ev

Received September 13, 2004

Abstract—In the theory of formations of finite solvable groups, there is a well-known result due to Blessenohl claiming that, for any local formation $\mathfrak{F}$, the class of groups for which every Hall π-subgroup belongs to $\mathfrak{F}$ also is a local formation. In the present paper, we obtain a result exactly dual to that indicated in the theory of Fitting classes. We prove that if a Fitting class $\mathfrak{F}$ is local, then the class of all groups all of whose Hall π-subgroups belong to $\mathfrak{F}$ is also local.

Key words: *Hall subgroup, formation of finite groups, local formation, Fitting class, Hartley function, Lockett class, Sylow subgroup.*

INTRODUCTION

Several investigations of canonical subgroups of finite solvable groups are related to the study of classes (of finite groups) defined by given properties of Hall subgroups. In the theory of formations, the construction of the class $B_\pi(\mathfrak{X})$ of all groups all of whose Hall π-subgroups belong to a given local formation $\mathfrak{X}$ is well known in connection with its applications to the study of the properties of Hall subgroups. This is caused first of all by Blessenohl's result [1] implying that the class $B_\pi(\mathfrak{X})$ is a local formation for any local formation $\mathfrak{X}$. In the theory of Fitting classes, Hauck [2] introduced a similar construction of the class $K_\pi(\mathfrak{F})$ of all groups all of whose Hall π-subgroups belong to the Fitting class $\mathfrak{F}$. As is known (see, e.g., [3, Chap. 9, Sec. 1.24]), a class of this kind is a Fitting class. Later on, Brison [4] described the $\mathfrak{F}$-radicals of the Hall π-subgroups in terms of the class $K_\pi(\mathfrak{F})$. However, the problem of whether the Fitting class $K_\pi(\mathfrak{F})$ is local or not remained open. In the present paper, we dualize the above Blessenohl result, namely, we prove that the class $K_\pi(\mathfrak{F})$ is local for any solvable local Fitting class $\mathfrak{F}$. Moreover, we prove that this result remains valid for partially local Fitting classes as well.

Recall that if π is a set of primes, then the symbol G_π denotes a Hall π-subgroup of G, i.e., a subgroup whose order is a π-number and the index is a π'-number.

A class of groups $\mathfrak{F}$ is called a *Fitting class* if $\mathfrak{F}$ is closed with respect to taking normal subgroups and products of normal $\mathfrak{F}$-subgroups. It follows from the definition that, for any nonempty Fitting class $\mathfrak{F}$, any group G admits a unique $\mathfrak{F}$-maximal normal subgroup $G_\mathfrak{F}$ of G. This subgroup is referred to as the $\mathfrak{F}$-*radical* of G. Denote by $\mathfrak{F}\mathfrak{H}$ the product of the Fitting classes $\mathfrak{F}$ and $\mathfrak{H}$, i.e., the class of all groups G for which $G/G_\mathfrak{F} \in \mathfrak{H}$. As is well known, the product of Fitting classes is a Fitting class, and the multiplication operation for Fitting classes is associative.

We shall use the concept of Shemetkov–Skiba partial localization [5], which is as follows.

Let $\varnothing \neq w \subseteq \mathbb{P}$, where $\mathbb{P}$ is the set of all prime numbers and $w' = \mathbb{P} \setminus w$. Every mapping

$$f\colon w \cup \{w'\} \to \{\text{the Fitting classes}\}$$

is referred to as a *w-local Hartley function* or a *w-local H-function* [5]. For any w-local H-function f, we introduce the support of f by the rule

$$\mathrm{Supp}(f) = \{a \in w \cup \{w'\} \mid f(a) \neq \varnothing\}.$$

Following [5], we set

$$LR_w(f) = \Big(\bigcap_{p\in\pi_2} \mathfrak{S}_{p'}\Big) \cap \Big(\bigcap_{p\in\pi_1} f(p)\mathfrak{N}_p\mathfrak{S}_{p'}\Big) \cap f(w')\mathfrak{S}_w,$$

where $\pi_1 = \mathrm{Supp}(f) \cap w$, $\pi_2 = w \setminus \pi_1$, and $\mathfrak{S}_w$ is the class of all solvable w-groups.

The Fitting class $\mathfrak{F}$ is said to be *w-local* [5] if $\mathfrak{F} = LR_w(f)$ for some w-local H-function f.

Note that, in the case of $w = \mathbb{P}$, any w-local Fitting class is said to be *local* and any w-local Hartley function is said to be a *local Hartley function* or a *local H-function*.

In the paper, we consider finite and solvable groups only.

Other definitions and notation can be found in [3, 6] if necessary.

1. PROPERTIES OF THE CLASS $K_\pi(\mathfrak{F})$

Recall that if $\mathfrak{F}$ is a Fitting class and π is a set of primes, then the symbol $K_\pi(\mathfrak{F})$ denotes the class of all groups in which every Hall π-subgroup is an $\mathfrak{F}$-group, i.e.,

$$K_\pi(\mathfrak{F}) = (G \in \mathfrak{S} : G_\pi \in \mathfrak{F}).$$

If $\mathfrak{F} = \varnothing$, then we set $K_\pi(\mathfrak{F}) = \varnothing$. If $\pi = \varnothing$ and $\pi = \mathbb{P}$, then we set $K_\varnothing(\mathfrak{F}) = \mathfrak{S}$ and $K_\mathbb{P}(\mathfrak{F}) = \mathfrak{F}$, respectively.

We shall repeatedly make use of certain known properties of the class $K_\pi(\mathfrak{F})$. These properties are listed in the following lemma.

Lemma 1.1 [4]**.** *Let $\mathfrak{F}$ and $\mathfrak{X}$ be Fitting classes, let π be a set of primes, let G be a group, and let G_π be a Hall π-subgroup of G. Then*

(1) $K_\pi(\mathfrak{F} \cap \mathfrak{X}) = K_\pi(\mathfrak{F}) \cap K_\pi(\mathfrak{X})$ *and, if* $\mathfrak{F} \subseteq \mathfrak{X}$, *then* $K_\pi(\mathfrak{F}) \subseteq K_\pi(\mathfrak{X})$;

(2) *if $\mathfrak{F}$ is a nonempty class, then* $G_{K_\pi(\mathfrak{F})} \cap G_\pi = (G_\pi)_\mathfrak{F}$;

(3) $K_\pi(\mathfrak{F}\mathfrak{X}) = K_\pi(\mathfrak{F})K_\pi(\mathfrak{X})$.

Lemma 1.2. *Let $\mathfrak{F}$ be a Fitting class, and let π be a set of primes. Then*

1) *if $p \in \pi$, then* $K_\pi(\mathfrak{S}_{p'}) = \mathfrak{S}_{p'}$;

2) *if $\mathfrak{F} \neq \varnothing$ and $\mathfrak{F}\mathfrak{N}_p = \mathfrak{F}$ for some prime p, then* $K_\pi(\mathfrak{F})\mathfrak{N}_p = K_\pi(\mathfrak{F})$.

Proof. Obviously, $\mathfrak{S}_{p'} \subseteq K_\pi(\mathfrak{S}_{p'})$. Let $G \in K_\pi(\mathfrak{S}_{p'})$, and let G_π be a Hall π-subgroup of G. Then $|G_\pi|$ is a p'-number. However, $p \in \pi$, and, therefore, $\pi' \subseteq p'$. Hence $G \in \mathfrak{S}_{p'}$ and $K_\pi(\mathfrak{S}_{p'}) \subseteq \mathfrak{S}_{p'}$. This proves assertion (1).

Let us prove assertion (2). Let $G \in K_\pi(\mathfrak{F})\mathfrak{N}_p$. Since $G_\pi \cap G_{K_\pi(\mathfrak{F})} = (G_\pi)_\mathfrak{F}$ by assertion (2) of Lemma 1.1, it follows from the isomorphism

$$G_\pi G_{K_\pi(\mathfrak{F})}/G_{K_\pi(\mathfrak{F})} \cong G_\pi/(G_\pi \cap G_{K_\pi(\mathfrak{F})}) = G_\pi/(G_\pi)_\mathfrak{F}$$

that

$$G_\pi/(G_\pi)_\mathfrak{F} \cong G_\pi G_{K_\pi(\mathfrak{F})}/G_{K_\pi(\mathfrak{F})}.$$

However, the Hall π-subgroup $G_\pi G_{K_\pi(\mathfrak{F})}/G_{K_\pi(\mathfrak{F})}$ of the group $G/G_{K_\pi(\mathfrak{F})}$ is a p-group. Hence $G_\pi/(G_\pi)_\mathfrak{F} \in \mathfrak{N}_p$ and $G_\pi \in \mathfrak{F}$, and we have the inclusion

$$K_\pi(\mathfrak{F})\mathfrak{N}_p \subseteq K_\pi(\mathfrak{F}).$$

The converse inclusion is obvious. This proves the lemma. □

Lemma 1.3. *Let $\mathfrak{F}$ be a nonempty Fitting class, and let π and σ be sets of prime numbers such that $\sigma \cap \pi = \varnothing$. Then*

$$K_\sigma(\mathfrak{F})\mathfrak{S}_\pi = K_\sigma(\mathfrak{F}).$$

Proof. It is clear that $K_\sigma(\mathfrak{F}) \subseteq K_\sigma(\mathfrak{F})\mathfrak{S}_\pi$.

Let $G \in K_\sigma(\mathfrak{F})\mathfrak{S}_\pi$. By assertion (2) of Lemma 1.1, we have $G_\sigma \cap G_{K_\sigma(\mathfrak{F})} = (G_\sigma)_{\mathfrak{F}}$. Using the isomorphism

$$G_\sigma G_{K_\sigma(\mathfrak{F})}/G_{K_\sigma(\mathfrak{F})} \cong G_\sigma/(G_\sigma \cap G_{K_\sigma(\mathfrak{F})}) = G_\sigma/(G_\sigma)_{\mathfrak{F}},$$

we obtain

$$G_\sigma/(G_\sigma)_{\mathfrak{F}} \cong G_\sigma G_{K_\sigma(\mathfrak{F})}/G_{K_\sigma(\mathfrak{F})}.$$

In this case, it follows from the relation $G/G_{K_\sigma(\mathfrak{F})} \in \mathfrak{S}_\pi$ that $G_\sigma/(G_\sigma)_{\mathfrak{F}} \in \mathfrak{S}_\pi$, and, therefore, $G_\sigma = (G_\sigma)_{\mathfrak{F}}$. Hence $G_\sigma \in \mathfrak{F}$ and $G \in K_\sigma(\mathfrak{F})$. This proves the lemma. □

Corollary 1.4. *Let $\mathfrak{F}$ be a nonempty Fitting class, and let π and σ be sets of prime numbers such that $\sigma \cap \pi = \varnothing$. Then*

$$K_\sigma(\mathfrak{F})\mathfrak{N}_\pi = K_\sigma(\mathfrak{F}).$$

Proof. The proof follows from the relation

$$K_\sigma(\mathfrak{F})\mathfrak{N}_\pi \subseteq K_\sigma(\mathfrak{F})\mathfrak{S}_\pi = K_\sigma(\mathfrak{F}),$$

which holds by Lemma 1.3. □

Corollary 1.5. *If $\mathfrak{F}$ is a nonempty Fitting class and π is a set of prime numbers, then*

$$K_\pi(\mathfrak{F})\mathfrak{S}_{\pi'} = K_\pi(\mathfrak{F}).$$

2. LOCAL PROPERTY OF THE CLASS $K_\pi(\mathfrak{F})$

The main result of the present paper is the following theorem.

Theorem 2.1. *If $\mathfrak{F}$ is a w-local Fitting class and if π is a set of prime numbers, then $K_{\pi\cap w}(\mathfrak{F})$ is a w-local Fitting class.*

Proof. Suppose that $\mathfrak{F} = \varnothing$. Since the Fitting class $\varnothing$ is w-local (see, [5, Example 10]), it follows that the $K_{\pi\cap w}(\varnothing)$ is w-local, and the theorem holds in this case.

Suppose that $\mathfrak{F} \neq \varnothing$. If $w \cap \pi = \varnothing$, then $K_\varnothing(\mathfrak{F}) = \mathfrak{S}$ by definition. Introduce an H-function f as follows: $f(a) = \mathfrak{S}$ for any a in $w \cup \{w'\}$. Then we clearly have

$$LR_w(f) = \left(\bigcap_{p\in w} \mathfrak{S}\mathfrak{N}_p\mathfrak{S}_{p'}\right) \cap \mathfrak{S}\mathfrak{S}_w = \mathfrak{S},$$

and the Fitting class $K_{\pi\cap w}(\mathfrak{F})$ is w-local.

Suppose that $w \cap \pi \neq \varnothing$. Since $\mathfrak{F}$ is a w-local Fitting class, it follows from Theorem 9 in [5] that there is a w-local H-function F such that $\mathfrak{F} = LR_w(F)$, $F(p) = F(p)\mathfrak{N}_p \subseteq \mathfrak{F}$ for any p in w, and $F(w') = \mathfrak{F}$.

Write

$$\pi_1 = \mathrm{Supp}(F) \cap w, \qquad \pi_2 = w \setminus \pi_1.$$

In this case,

$$\mathfrak{F} = LR_w(F) = \left(\bigcap_{p\in\pi_2} \mathfrak{S}_{p'}\right) \cap \left(\bigcap_{p\in\pi_1} F(p)\mathfrak{N}_p\mathfrak{S}_{p'}\right) \cap \mathfrak{F}\mathfrak{S}_w.$$

Let us construct a w-local H-function as follows:

$$f(p)=\begin{cases} K_{\pi_1\cap\pi}(F(p)) & \text{if } p\in\pi_1\cap\pi\,;\\ K_{\pi\cap w}(\mathfrak{F}) & \text{if } p\in w\setminus\pi\,;\\ \varnothing & \text{if } p\in\pi_2\cap\pi\,, \end{cases}$$

and $f(w')=K_{\pi\cap w}(\mathfrak{F})$.

It can readily be seen that $\operatorname{Supp}(f)=(\pi_1\cap\pi)\cup(w\setminus\pi)\cup\{w'\}$. Hence

$$LR_w(f)=\Bigl(\bigcap_{p\in\pi_2\cap\pi}\mathfrak{S}_{p'}\Bigr)\cap\Bigl(\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p\mathfrak{S}_{p'}\Bigr)$$
$$\cap\Bigl(\bigcap_{p\in w\setminus\pi}K_{\pi\cap w}(\mathfrak{F})\mathfrak{N}_p\mathfrak{S}_{p'}\Bigr)\cap K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_w.$$

Let us first show that

$$\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p\mathfrak{S}_{p'}=K_{\pi_1\cap\pi}(\mathfrak{F}). \tag{1}$$

Since $\mathfrak{F}$ is a w-local Fitting class and $F(p)=F(p)\mathfrak{N}_p$ for any p in w, we have the inclusion $\mathfrak{F}\subseteq\bigcap_{p\in\pi_1\cap\pi}F(p)\mathfrak{S}_{p'}$. In this case,

$$K_{\pi_1\cap\pi}(\mathfrak{F})\subseteq K_{\pi_1\cap\pi}\Bigl(\bigcap_{p\in\pi_1\cap\pi}F(p)\mathfrak{S}_{p'}\Bigr)=\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p)\mathfrak{S}_{p'})$$

by assertion (1) of Lemma 1.1. Hence, by assertion (3) of Lemma 1.1 and by assertion (1) of Lemma 1.2, we obtain

$$\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p)\mathfrak{S}_{p'})=\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p))\mathfrak{S}_{p'}.$$

Moreover, by assertion (2) of Lemma 1.2, for any $p\in\pi_1\cap\pi$ we have

$$K_{\pi_1\cap\pi}(F(p))\subseteq K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p.$$

Thus,

$$K_{\pi_1\cap\pi}(\mathfrak{F})\subseteq\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p\mathfrak{S}_{p'}.$$

Let us prove the converse inclusion. Indeed, since $F(p)\subseteq\mathfrak{F}$ for any p in w, it follows from assertion (1) of Lemma 1.1 that

$$K_{\pi_1\cap\pi}(F(p))\subseteq K_{\pi_1\cap\pi}(\mathfrak{F}).$$

Thus,

$$K_{\pi_1\cap\pi}(F(p))\mathfrak{S}_{p'}\subseteq K_{\pi_1\cap\pi}(\mathfrak{F})\mathfrak{S}_{p'}$$

and, by assertion (2) of Lemma 1.2, we have

$$K_{\pi_1\cap\pi}(F(p))=K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p$$

for any p in $\pi_1\cap\pi$. Hence

$$\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p\mathfrak{S}_{p'}\subseteq\bigcap_{p\in\pi_1\cap\pi}K_{\pi_1\cap\pi}(\mathfrak{F})\mathfrak{S}_{p'}=K_{\pi_1\cap\pi}(\mathfrak{F})\mathfrak{S}_{(\pi_1\cap\pi)'}.$$

Moreover, by Corollary 1.5,

$$K_{\pi_1\cap\pi}(\mathfrak{F})\mathfrak{S}_{(\pi_1\cap\pi)'} = K_{\pi_1\cap\pi}(\mathfrak{F}).$$

Thus,

$$\bigcap_{p\in\pi_1\cap\pi} K_{\pi_1\cap\pi}(F(p))\mathfrak{N}_p\mathfrak{S}_{p'} \subseteq K_{\pi_1\cap\pi}(\mathfrak{F}),$$

and this proves relation (1).

Let us now prove that

$$\bigcap_{p\in w\setminus\pi} K_{\pi\cap w}(\mathfrak{F})\mathfrak{N}_p\mathfrak{S}_{p'} = K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{(w\setminus\pi)'}. \tag{2}$$

By Lemma 1.3, $K_{\pi\cap w}(\mathfrak{F}) = K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{w\setminus\pi}$. Moreover, $\mathfrak{S}_{w\setminus\pi}\mathfrak{N}_p = \mathfrak{S}_{w\setminus\pi}$ for any p in $w\setminus\pi$. Hence

$$\bigcap_{p\in w\setminus\pi} K_{\pi\cap w}(\mathfrak{F})\mathfrak{N}_p\mathfrak{S}_{p'} = \bigcap_{p\in w\setminus\pi} K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{w\setminus\pi}\mathfrak{N}_p\mathfrak{S}_{p'} = \bigcap_{p\in w\setminus\pi} K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{w\setminus\pi}\mathfrak{S}_{p'}$$

and

$$\bigcap_{p\in w\setminus\pi} K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{w\setminus\pi}\mathfrak{S}_{p'} = \bigcap_{p\in w\setminus\pi} K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{p'} = K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{(w\setminus\pi)'},$$

which proves relation (2).

Thus, we have proved the validity of the relation

$$LR_w(f) = \mathfrak{S}_{(\pi_2\cap\pi)'} \cap K_{\pi_1\cap\pi}(\mathfrak{F}) \cap K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_{(w\setminus\pi)'} \cap K_{\pi\cap w}(\mathfrak{F})\mathfrak{S}_w. \tag{3}$$

With regard to (3), to prove the theorem, it now suffices to justify the relation

$$\mathfrak{S}_{(\pi_2\cap\pi)'} \cap K_{\pi_1\cap\pi}(\mathfrak{F}) = K_{\pi\cap w}(\mathfrak{F}). \tag{4}$$

Let $G \in K_{\pi\cap w}(\mathfrak{F})$. Then it follows from $\mathfrak{F} = LR_w(F)$ that $G_{\pi\cap w} \in \mathfrak{F} \subseteq \mathfrak{S}_{\pi_2'}$, and, therefore, $|G_{\pi\cap w}|$ is a $(\pi_2'\cap\pi\cap w)$-number. Moreover, $|G : G_{\pi\cap w}|$ is a $(\pi\cap w)'$-number. It can readily be seen that $(\pi\cap w)' \cup (\pi_2'\cap\pi\cap w) = (\pi_2\cap\pi)'$, and thus $G \in \mathfrak{S}_{(\pi_2\cap\pi)'}$. Since $\pi_1 \subseteq w$, we have $(\pi\cap w)' \subseteq (\pi_1\cap\pi)'$, and $|G : G_{\pi\cap w}|$ is a $(\pi_1\cap\pi)'$-number. Further, it follows from the relation $\pi_1\cap\pi\cap w \subseteq \pi_1\cap\pi$ that $|G_{\pi\cap w}|$ is a $(\pi_1\cap\pi)$-number. In this case, every $(\pi\cap w)$-Hall subgroup $G_{\pi\cap w}$ of G is also a $(\pi_1\cap\pi)$-Hall subgroup of G. Thus, $G_{\pi_1\cap\pi} \in \mathfrak{F}$ and $G \in K_{\pi_1\cap\pi}(\mathfrak{F})$. Hence

$$K_{\pi\cap w}(\mathfrak{F}) \subseteq \mathfrak{S}_{(\pi_2\cap\pi)'} \cap K_{\pi_1\cap\pi}(\mathfrak{F}).$$

Let us prove the converse inclusion. Suppose that $G \in \mathfrak{S}_{(\pi_2\cap\pi)'} \cap K_{\pi_1\cap\pi}(\mathfrak{F})$. Then $G_{\pi_1\cap\pi} \in \mathfrak{F}$. Since $(\pi_2\cap\pi)' \setminus (\pi_1\cap\pi) = (\pi\cap w)'$, it follows that $|G : G_{\pi_1\cap\pi}|$ is a $(\pi\cap w)'$-number. Now, since $\pi_1\cap\pi \subseteq \pi\cap w$, we see that $|G_{\pi_1\cap\pi}|$ is a $(\pi\cap w)$-number. Thus, every $(\pi_1\cap\pi)$-Hall subgroup of G is a $(\pi\cap w)$-Hall subgroup of G, and, therefore, $G_{\pi\cap w} \in \mathfrak{F}$. Hence $G \in K_{\pi\cap w}(\mathfrak{F})$. This means that

$$\mathfrak{S}_{(\pi_2\cap\pi)'} \cap K_{\pi_1\cap\pi}(\mathfrak{F}) \subseteq K_{\pi\cap w}(\mathfrak{F}),$$

which proves relation (4).

This completes the proof of the theorem. $\square$

For $w = \mathbb{P}$, the theorem implies a result which is exactly dual to Blessenohl's result [1] mentioned in the introduction.

Corollary 2.2. *If $\mathfrak{F}$ is a local Fitting class and π is a set of prime numbers, then $K_\pi(\mathfrak{F})$ is a local Fitting class.*

It follows from the corollary that the family of all Fitting classes $\mathfrak{F}$ for which $K_\pi(\mathfrak{F})$ is local is large because it contains all local Fitting classes. However, in the general case, the Fitting class $K_\pi(\mathfrak{F})$ need not be local for a given Fitting class $\mathfrak{F}$. This is justified by the following example.

Example 2.3. Let $\mathfrak{F}$ be an arbitrary nontrivial (distinct from the classes (1) and $\mathfrak{S}$) normal Fitting class. Let us show that $\mathfrak{F}$ is not local. Indeed, if $\mathfrak{F}$ is local, then $\mathfrak{F}$ is a Lockett class by Lemma 5 in [7], and, therefore, $\mathfrak{F}^* = \mathfrak{F} = \mathfrak{S}$ by [3, Theorem X.3.7]. The contradiction thus obtained shows that every nontrivial normal Fitting class is nonlocal.

Let now p and q be prime numbers such that $p \mid (q-1)$, and let $G = D^p_{q^n}$ be a monolithic group with normal Abelian Sylow q-subgroup of exponent q^n and a cyclic Hall q'-subgroup of order p. Let $\pi = \pi(G)$, and let $\mathfrak{S}_*$ be a minimal normal Fitting class. In this case, by Berger's result (see property 3 in [8]), we have $G \notin \mathfrak{S}_*$, and, therefore, $G \notin K_\pi(\mathfrak{S}_*)$. Hence $K_\pi(\mathfrak{S}_*) \neq \mathfrak{S}$, and the Fitting class $K_\pi(\mathfrak{S}_*)$ is nontrivial. However, by Hauck's theorem (see [2, Theorem 3.4]), $K_\pi(\mathfrak{S}_*)$ is a normal Fitting class, and hence it is nonlocal.

In conclusion we note that if $\mathfrak{F}$ is a nonempty w-local Fitting class and π is a set of prime numbers, where $w \cap \pi = \varnothing$, then the class $K_\pi(\mathfrak{F})$ is also w-local. Indeed, in this case, defining the values of a w-local H-function by the relation $f(a) = K_\pi(\mathfrak{F})$ for any a in $w \cup \{w'\}$, we obtain

$$LR_w(f) = \left(\bigcap_{p\in w} K_\pi(\mathfrak{F})\mathfrak{N}_p\mathfrak{S}_{p'}\right) \cap K_\pi(\mathfrak{F})\mathfrak{S}_w = K_\pi(\mathfrak{F})\mathfrak{N}_w\mathfrak{S}_{w'} \cap K_\pi(\mathfrak{F})\mathfrak{S}_w.$$

Using Corollary 1.4, we now see that $K_\pi(\mathfrak{F})\mathfrak{N}_w = K_\pi(\mathfrak{F})$ and

$$LR_w(f) = K_\pi(\mathfrak{F})\mathfrak{S}_{w'} \cap K_\pi(\mathfrak{F})\mathfrak{S}_w = K_\pi(\mathfrak{F})(\mathfrak{S}_{w'} \cap \mathfrak{S}_w) = K_\pi(\mathfrak{F}).$$

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P. M. Masherov Vitebsk State University (Belarus)
E-mail: (V. N. Zagurskii) zagurski@yandex.ru
(N. T. Vorob′ev) nicholas@vsu.by