RADICAL CLASSES OF FINITE GROUPS WITH THE LOCKETT CONDITION

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In the theory of radical classes of finite solvable groups (Fitting classes) the following is a well-known problem (the Lockett conjecture): can every radical class $\mathfrak{F}$ be obtained as intersection of two radical classes $\mathfrak{F}^*$ and $\mathfrak{X}$, where $\mathfrak{F}^* = \{G \in \mathfrak{S} \mid (G^2)_{\mathfrak{F}} = (G_{\mathfrak{F}})^2 \langle g, g^{-1} \rangle \mid g \in G\}$ and $\mathfrak{X}$ is some normal radical class [1]?

First results on this problem were obtained by Bryce and Cossey [2] who proved that the Lockett conjecture holds for all radical classes $\mathfrak{F}$, satisfying $\mathfrak{F}_* = \mathfrak{F}^* \cap \mathfrak{S}_*$; here $\mathfrak{F}_* = \bigcap \{\mathfrak{H} \mid \mathfrak{H}^* = \mathfrak{F}^*\}$ and $\mathfrak{S}_*$ is a minimal radical class. A radical class of groups $\mathfrak{F}$ for which $\mathfrak{F}_* = \mathfrak{F}^* \cap \mathfrak{S}_*$ is said to satisfy the Lockett condition [3]. Shortly after, an example was constructed in [4] of a radical class $\mathfrak{F}$ such that $\mathfrak{F}_* \neq \mathfrak{F}^* \cap \mathfrak{S}_*$; however, the problem of describing the radical classes satisfying the Lockett condition remains open (cf. [2, 3]).

At present the most general result in this direction is the result of Beidleman and Hauck [3] which says that radical classes of the form $\mathfrak{X}\mathfrak{S}_{\pi}\mathfrak{S}_{\pi'}$, $\mathfrak{X}\mathfrak{N}$ satisfy the Lockett condition.

In [5] the author announced results relating to the study of local radical classes; these are constructions which are dual to well-known basic objects in the theory of formations of finite groups, viz. local formations (cf [6, Chap. 1]).

The main result of the present paper is to prove that every local radical class of finite solvable groups satisfies the Lockett condition. The previously quoted results [3] and [2] are special cases of our theorem (radical classes of the form $\mathfrak{X}\mathfrak{S}_{\pi}\mathfrak{S}_{\pi''}$, $\mathfrak{X}\mathfrak{N}$ are examples of local classes). From the main result we obtain also an easy positive answer to question 8.30 posed by Lausch in the Kourovska Notebook [7] for the case of local radical classes (Corollary 2).

If $\mathfrak{F}$ is a radical class then $G_{\mathfrak{F}}$ denotes the $\mathfrak{F}$ -radical of the group G, $\mathfrak{F}\mathfrak{H}$ denotes the product of the radical classes $\mathfrak{F}$ and $\mathfrak{H}$, i.e., the class of all groups G for which $G/G_{\mathfrak{F}} \in \mathfrak{H}$. A radical class is called local [5] if there exists a local group function f [6] such that f(p) is a radical class for all prime numbers p and $\mathfrak{F} = \bigcap_p f(p)\, \mathfrak{S}_p \mathfrak{S}_{p'}$. In this case we will call f a radical function of the class $\mathfrak{F}$. A radical class $\mathfrak{F}$ is called a Lockett class [1] if $\mathfrak{F} = \mathfrak{F}^*$. Other definitions and notation can be found in [6, 8] and [9] if required. All groups under consideration are finite and solvable. The following definition is due to L. A. Shemetkov.

Definition 1. Let f be a radical function of the class $\mathfrak{F}$. Then we say that

1) f is inner if $f(p) \subseteq \mathfrak{F}$ for all primes p;

2) f is complete if $f(p)\mathfrak{S}_p = f(p)$ for all primes p.

LEMMA 1. Every local radical class (in the general case of not necessarily solvable groups) is determined by a complete inner radical function.

Proof. Assume that $\mathfrak{F}$ is a local radical class. Then there exists a radical function f such that $\mathfrak{F} = \bigcap_p f(p)\, \mathfrak{S}_p \mathfrak{S}_{p'}$. We define a radical function φ as follows: $\varphi(p) = f(p) \cap \mathfrak{F}$ for every prime p. It is clear that $\mathfrak{F} = \bigcap_p \varphi(p) \mathfrak{S}_p \mathfrak{S}_{p'}$.

Now we define the radical function ψ such that $\psi(p) = \varphi(p)\mathfrak{S}_p$. It is clear that ψ is a complete radical function of the class $\mathfrak{F}$. We will now show that ψ is an inner radical function. Let $G \in \psi(p)$. Then $G/G_{\varphi(p)} \in \mathfrak{S}_p \mathfrak{S}_{p'}$ and therefore $G \in \varphi(p)\mathfrak{S}_p\mathfrak{S}_{p'}$. Assume now that q is any prime number different from p. Then $\mathfrak{S}_p \subseteq \mathfrak{S}_{q'}$ and therefore $G^{\mathfrak{S}_{q'}} \in \mathfrak{F}$. It follows that $(G^{\mathfrak{S}_{q'}})^{\mathfrak{S}_q \mathfrak{S}_{q'}} \in \varphi(q)$. Using Theorem 7 of [9] we obtain $G^{\mathfrak{S}_q \mathfrak{S}_{q'}} \in \varphi(q)$. Therefore $G/G_{\varphi(q)} \simeq G/G^{\mathfrak{S}_q\mathfrak{S}_{q'}}/G_{\varphi(q)}/G^{\mathfrak{S}_q\mathfrak{S}_{q'}} \in \mathfrak{S}_q\mathfrak{S}_{q'}$. We have shown that $G \in \bigcap_p \varphi(p)\, \mathfrak{S}_p \mathfrak{S}_{p'} = \mathfrak{F}$.

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The Lemma is established.

Definition 2. Let $\mathfrak{F}$, $\mathfrak{H}$ be radical classes of groups. By $\mathfrak{F} \circ \mathfrak{H}$ we denote the class of groups defined as follows:

$G \in \mathfrak{F} \circ \mathfrak{H}$ if and only if the $\mathfrak{F}$-injector of G belongs to $\mathfrak{H}$.

It is easy to see that the class $\mathfrak{F} \circ \mathfrak{H}$ is S_n-closed, but in general it is not a radical class (cf. example 3.2 (b) [10]). The construction of this class of groups is dual to one known in the theory of formations, namely the formation product of second kind; this has been studied in [11, 12].

Recall that a radical class $\mathfrak{F}$ is called a Fischer class [1] if $K \subseteq H \subseteq G \in F$, $K \trianglelefteq G$ and $H/K \in \mathfrak{S}_p$ always implies $H \in \mathfrak{F}$ (p is some prime number).

In the following three lemmas we do not assume that the groups under consideration are solvable.

LEMMA 2. Every local radical class is a Fischer class.

Proof. Let f be a radical function of the class $\mathfrak{F}$ and G a group from $\mathfrak{F}$. We will show that if K is a normal subgroup of G such that $K \subseteq H \subseteq G$ and $H/K \in \mathfrak{S}_q$ for some prime number q then $H \in \mathfrak{F}$. It will be sufficient to show that $H^{\mathfrak{S}_p \mathfrak{S}_{p'}} \in f(p)$ for all prime numbers p. We distinguish two possibilities.

1. p is a prime number different from q.

In this case $H/K \in \mathfrak{S}_{p'}$ and therefore $H^{\mathfrak{S}_{p'}} \subseteq K$. Hence $K^{\mathfrak{S}_{p'}} = H^{\mathfrak{S}_{p'}}$ and therefore $K^{\mathfrak{S}_p \mathfrak{S}_{p'}} = H^{\mathfrak{S}_p \mathfrak{S}_{p'}}$. But $K \in \mathfrak{F}$ and therefore $H^{\mathfrak{S}_p \mathfrak{S}_{p'}} \in f(p)$.

2. p = q. Let H_q be a Sylow q-subgroup of the group H. Then H_qK/K is a Sylow q-subgroup of H/K. Consequently, $H = H_qK$. Since the Sylow q-subgroup G_q of G is contained in $G^{\mathfrak{S}_{q'}}$ and H_q is contained in G_q we have $[H_q, K] \subseteq [G^{\mathfrak{S}_{q'}}, K] \subseteq G^{\mathfrak{S}_{q'}} \cap K$. $G \in \mathfrak{F}$, hence $G^{\mathfrak{S}_{q'}} \subseteq G_{f(q)\mathfrak{S}_q}$. Therefore $[H_q, K] \subseteq K \cap G_{f(q)\mathfrak{S}_q} = K_{f(q)\mathfrak{S}_q} = R$. It is easy to see that $RH_q \trianglelefteq KH_q = H$. Since $RH_q = f(q)\,\mathfrak{S}_q$, it follows from $H/RH_q \in \mathfrak{S}_{q'}$ that $H \in f(q)\,\mathfrak{S}_q\mathfrak{S}_{q'}$. Hence $H \in \cap_p f(p)\,\mathfrak{S}_p\mathfrak{S}_{p'} = \mathfrak{F}$.

The Lemma is established.

A radical class of groups which is at the same time a homomorph (formation), closed under the operation Ext_{Φ} (cf. also [6, p. 12]) will be called a saturated radical homomorph (respectively, a saturated radical formation).

LEMMA 3. If $\mathfrak{F}$ is some radical class and $\mathfrak{H}$ a saturated radical homomorph then

$$\mathfrak{F}^*\mathfrak{H} = (\mathfrak{F}\mathfrak{H})^*.$$

Proof. We prove first that $\mathfrak{F}^*\mathfrak{H} \subseteq (\mathfrak{F}\mathfrak{H})^*$. Assume the contrary. Now choose in the class $\mathfrak{F}^*\mathfrak{H} \setminus (\mathfrak{F}\mathfrak{H})^*$ a group G of minimal order. It is clear that $G_{\mathfrak{F}} \subseteq G_{\mathfrak{F}^*}$. Assume that $G_{\mathfrak{F}^*}/G_{\mathfrak{F}} \subseteq \Phi(G/G_{\mathfrak{F}})$. Since $\mathfrak{H}$ is a homomorph it follows from $G/G_{\mathfrak{F}^*} \in \mathfrak{H}$ that $G/G_{\mathfrak{F}}/G_{\mathfrak{F}^*}/G_{\mathfrak{F}} \in \mathfrak{H}$. Consequently, $G/G_{\mathfrak{F}}/\Phi(G/G_{\mathfrak{F}}) \in \mathfrak{H}$ and therefore $G/G_{\mathfrak{F}} \in \mathfrak{H}$.

Thus $G \in (\mathfrak{F}\mathfrak{H})^*$, which is impossible.

If $G_{\mathfrak{F}^*}/G_{\mathfrak{F}}$ is not contained in $\Phi(G/G_{\mathfrak{F}})$ then $G/G_{\mathfrak{F}}$ contains a maximal subgroup $M/G_{\mathfrak{F}}$ such that $G/G_{\mathfrak{F}} = (G_{\mathfrak{F}^*}/G_{\mathfrak{F}})(M/G_{\mathfrak{F}})$. By Theorem 3.5 1) [13] we have $G_{\mathfrak{F}^*}/G_{\mathfrak{F}} \subseteq Z(G/G_{\mathfrak{F}})$, hence $M/G_{\mathfrak{F}} \triangleleft G/G_{\mathfrak{F}}$. It now follows by induction that $M \in (\mathfrak{F}\mathfrak{H})^*$. Since $\mathfrak{F}^* \subseteq (\mathfrak{F}\mathfrak{H})^*$ we have $G_{\mathfrak{F}^*} \in (\mathfrak{F}\mathfrak{H})^*$. Therefore $G = G_{\mathfrak{F}^*}M \in (\mathfrak{F}\mathfrak{H})^*$. This contradiction shows that $\mathfrak{F}^*\mathfrak{H} \subseteq (\mathfrak{F}\mathfrak{H})^*$.

Now we shall prove the reverse inclusion. If $G \in \mathfrak{F}\mathfrak{H}$, it follows from the fact that $\mathfrak{H}$ is a homomorph that $G \in \mathfrak{F}^*\mathfrak{H}$. Hence $\mathfrak{F}\mathfrak{H} \subseteq \mathfrak{F}^*\mathfrak{H}$ and therefore by Theorem 3.3 c) of [13] we have $(\mathfrak{F}\mathfrak{H})^* \subseteq (\mathfrak{F}^*\mathfrak{H})^*$. By the definition of the class $\mathfrak{H}^*$ (cf. [1] and [13]) every group G of $\mathfrak{H}^*$ is a homomorphic image of the group $(G \times G)_{\mathfrak{H}}$, and therefore $G \in \mathfrak{H}$. It is therefore clear that $\mathfrak{H}$ is a Lockett class, i.e. $\mathfrak{H} = \mathfrak{H}^*$. Moreover, by Theorem 3.3 c) of [13] the class $\mathfrak{F}^*$ is also a Lockett class. Therefore $\mathfrak{F}^*\mathfrak{H}$ is a product of Lockett classes, and it follows easily from the proof of Lemma 3.1 (b) of [14] that $\mathfrak{F}^*\mathfrak{H}$ is also a Lockett class. Hence $(\mathfrak{F}\mathfrak{H})^* \subseteq \mathfrak{F}^*\mathfrak{H}$.

The Lemma is established.

LEMMA 4. Let $\mathfrak{F}_1$, $\mathfrak{F}_2$ be radical classes of groups. Then the following assertions hold:

1) if $\mathfrak{X}$ is a radical homomorph and $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ then $\mathfrak{F}_1\mathfrak{X} \subseteq \mathfrak{F}_2\mathfrak{X}$;

2) if $\mathfrak{X}$ is a radical formation then $(\mathfrak{F}_1 \cap \mathfrak{F}_2)\mathfrak{X} = \mathfrak{F}_1\mathfrak{X} \cap \mathfrak{F}_2\mathfrak{X}$.

Proof. Let $\mathfrak{X}$ be a radical homomorph and $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$. Then $G/G_{\mathfrak{F}_2} \simeq G/G_{\mathfrak{F}_1}/G_{\mathfrak{F}_2}/G_{\mathfrak{F}_1} \in \mathfrak{X}$ and therefore $G \in \mathfrak{F}_2\mathfrak{X}$.

By 1) we have $(\mathfrak{F}_1 \cap \mathfrak{F}_2)\mathfrak{X} \subseteq \mathfrak{F}_1\mathfrak{X} \cap \mathfrak{F}_2\mathfrak{X}$.

If $G/G_{\mathfrak{F}_i} \in \mathfrak{X}$ then $G/\bigcap G_{\mathfrak{F}_i} \in \mathfrak{X}$. It is clear that $\bigcap G_{\mathfrak{F}_i} = G_{\bigcap \mathfrak{F}_i}$. Therefore $G \in (\bigcap \mathfrak{F}_i)\mathfrak{X}$ (i = 1, 2).

The lemma is established.

Definition 3. The radical function f is called a Lockett function if f(p) is a Lockett class for all prime numbers p.

LEMMA 5. Every local radical class can be defined by a Lockett function. In particular, every local radical class is a Lockett class.

Proof. Let φ be a radical function of the class $\mathfrak{F}$. Then $\mathfrak{F} = \bigcap_p \varphi(p)\mathfrak{S}_p\mathfrak{S}_{p'}$. It is easy to see that for every prime p the radical class $\varphi(p)\mathfrak{S}_p\mathfrak{S}_{p'}$ is local. It follows from Lemma 2 that $\varphi(p)\mathfrak{S}_p\mathfrak{S}_{p'}$ is a Fischer class and thus by Theorem 2.2 d) of [1] it is also a Lockett class. Now we apply Lemma 3 and find that $(\varphi(p))^*\mathfrak{S}_p\mathfrak{S}_{p'} = \varphi(p)\mathfrak{S}_p\mathfrak{S}_{p'}$ for all primes p. We now construct the radical function f as follows: $f(p) = (\varphi(p))^*$ for all primes p. By Theorem 2.2 b) of [1] f is then a Lockett function locally defining $\mathfrak{F}$. The fact that $\mathfrak{F}$ is a Lockett class follows immediately from property 2.3 b) in [1].

The Lemma is established.

COROLLARY. Every local radical class can be defined by a complete inner Lockett function.

Proof. By Lemma 1 every local radical class $\mathfrak{F}$ can be defined by a complete inner radical function ψ which can be obtained easily from the Lockett function f (which exists by Lemma 5), if it is assumed that $\psi(p) = (f(p) \cap \mathfrak{F})\mathfrak{S}_p$ for all primes p.

Definition 4. Let f be a radical function of the class $\mathfrak{F}$. The radical class $\mathfrak{H}$ is said to be closed under f-injectors if $\mathfrak{H} \subseteq f(p) \circ \mathfrak{H}$ for all primes p.

Examples. Let f be an inner radical function of the class $\mathfrak{F}$. Then the radical class $\mathfrak{H}$ is closed under f-injectors in each of the following cases:

1) f(p) is an S-closed radical class for all primes p, $\mathfrak{H}$ is minimal normal;

2) $\mathfrak{H} = \mathfrak{X}\mathfrak{Y}$ where $\mathfrak{X} \subseteq f(p)$ for all primes p, $\mathfrak{Y}$ is an S-closed radical class;

3) $\mathfrak{H} \supseteq f(p)$ for all primes p or $\mathfrak{H} \supseteq \mathfrak{F}$;

4) f(p) is normal for all primes p, $\mathfrak{H}$ arbitrary;

5) $\mathfrak{H}$ is S-closed.

Indeed, in case 1) f(p) is a Fischer class for all primes p, and therefore by Theorem 4.4 of [10] the class $\mathfrak{H}$ is closed under f-injectors.

We now consider case 2). Let $G \in \mathfrak{H}$ and F an f(p)-injector of the group G. Then $G/G_{\mathfrak{X}} \in \mathfrak{H}$. But $G_{\mathfrak{X}} \subseteq G_{f(p)} \subseteq F$, and therefore $F/G_{\mathfrak{X}} \in \mathfrak{Y}$ since the class $\mathfrak{Y}$ is S-closed. By Theorem 1.1 of [15] the radical class $\mathfrak{Y}$ is a formation. Consequently, $F \in \mathfrak{X}\mathfrak{Y} = \mathfrak{H}$. The cases 3)-5) are trivial.

LEMMA 6. If ψ is a complete inner radical function of the class $\mathfrak{F}$ and $\mathfrak{H}$ is a radical class which is closed under ψ-injectors then $\mathfrak{F} \cap \mathfrak{H}_* = (\mathfrak{F} \cap \mathfrak{H})_*$.

Proof. Since $\mathfrak{F} \cap \mathfrak{H} \subseteq \mathfrak{H}$ it follows from Theorem 3.5 of [2] that $(\mathfrak{F} \cap \mathfrak{H})_* \subseteq \mathfrak{H}_*$. Consequently, $(\mathfrak{F} \cap \mathfrak{H})_* \subseteq \mathfrak{F} \cap \mathfrak{H}_*$. Now we prove the reverse inclusion. Let G be a group in $\mathfrak{H}$ and F a $\psi(p)$-injector of the group G where p is a prime. Then $F \in \psi(p) \cap \mathfrak{H}$ and therefore $F \in \mathfrak{F} \cap \mathfrak{H}$. Then it follows from Lemma 3 and Theorem 2.2 b) of [1] that $F \in ((\mathfrak{F} \cap \mathfrak{H})_*\mathfrak{S}_{p'})^* = (\mathfrak{F} \cap \mathfrak{H})^*\mathfrak{S}_{p'}$. Hence $G \in f(p) = \psi(p) \circ ((\mathfrak{F} \cap \mathfrak{H})_*\mathfrak{S}_{p'})^*$. We have thus shown that $\mathfrak{H} \subseteq f(p)$.

By Theorem 3.3 in [10] the class f(p) is a radical class. By Theorem 3 of [3] we therefore have the equation

$$\psi(p) \circ ((\mathfrak{F} \cap \mathfrak{H})_*\mathfrak{S}_{p'})^* = (\psi(p) \circ (\mathfrak{F} \cap \mathfrak{H})_*\mathfrak{S}_{p'})^*.$$

But then it follows from Theorems 3.5 of [2] and 2.2 b) of [1] that $\mathfrak{H}_* \subseteq \psi(p) \circ (\mathfrak{F} \cap \mathfrak{H})_*\mathfrak{S}_{p'}$.

Therefore

$$\psi(p) \cap \mathfrak{H}_* \subseteq (\mathfrak{F} \cap \mathfrak{H})_* \mathfrak{S}_{p'}.$$

By Lemma 4 1) we have $(\psi(p) \cap \mathfrak{H}_*) \mathfrak{E}_{p'} \subseteq (\mathfrak{F} \cap \mathfrak{H})_* \mathfrak{S}_{p'}$. Now we use part 2) of Lemma 4 and obtain that $\psi(p) \mathfrak{E}_{p'} \cap \mathfrak{H}_* \mathfrak{S}_{p'} \subseteq (\mathfrak{F} \cap \mathfrak{H})_* \mathfrak{S}_{p'}$ for all primes p. Therefore $\bigcap_p (\psi(p) \mathfrak{E}_{p'} \cap \mathfrak{H}_* \mathfrak{S}_{p'}) \subseteq \bigcap_p (\mathfrak{F} \cap \mathfrak{H})_* \mathfrak{S}_{p'}$. But then $(\bigcap_p \psi(p) \mathfrak{S}_{p'}) \cap (\mathfrak{H}_* (\bigcap_p \mathfrak{S}_{p'})) \subseteq (\mathfrak{F} \cap \mathfrak{H})_* (\bigcap_p \mathfrak{S}_{p'})$. Hence $\mathfrak{F} \cap \mathfrak{H}_* \subseteq (\mathfrak{F} \cap \mathfrak{H})_*$.

The Lemma is established.

<u>THEOREM</u>. Every local radical class satisfies the Lockett condition.

<u>Proof</u>. Let ψ be a complete inner radical function of the class $\mathfrak{F}$ and $\mathfrak{H}$ the class of all solvable groups. Then it is clear that $\mathfrak{H}$ is closed under ψ-injectors and it follows from Lemma 6 that $\mathfrak{F}_* = \mathfrak{F} \cap \mathfrak{H}_*$.

<u>COROLLARY 1</u> (Bryce, Cossey, Beidleman, Hauck [2, 3]). Primitive saturated formations and radical classes of groups of the types $\mathfrak{X}\mathfrak{S}_\pi\mathfrak{S}_{\pi'}$ and $\mathfrak{X}\mathfrak{N}$ satisfy the Lockett condition.

<u>Proof</u>. Obviously, the class of groups $\mathfrak{X}\mathfrak{S}_\pi\mathfrak{S}_{\pi'}$ is an example of a local class since it is defined by a complete inner radical function ψ such that for every prime p we have

$$\psi(p) = \begin{cases} \mathfrak{X}\mathfrak{S}_\pi, & \text{if} \quad p \in \pi, \\ \mathfrak{X}\mathfrak{S}_\pi\mathfrak{S}_{\pi'}, & \text{if} \quad p \in \pi'. \end{cases}$$

Now the result follows immediately from the theorem and Corollary 2, 3 in [3].

<u>COROLLARY 2</u>. If $\mathfrak{F}, \mathfrak{H}$ are local radical classes then $\mathfrak{F} \cap \mathfrak{H}$ is a radical class satisfying the Lockett condition.

<u>Proof</u>. It is clear that the radical class $\mathfrak{F} \cap \mathfrak{H}$ is local; hence the result follows from the Theorem.

<u>COROLLARY 3</u>. Let $\mathfrak{F}, \mathfrak{H}$ be radical classes where $\mathfrak{H}$ is local. Then the product $\mathfrak{F}\mathfrak{H}$ of radical classes satisfies the Lockett condition.

<u>Proof</u>. It is easy to see that $\mathfrak{F}\mathfrak{H}$ is a local radical class defined by a radical function f such that $f(p) = \mathfrak{F} h(p)$ for all primes p where h is some radical function of the class $\mathfrak{H}$. Now the result follows immediately from the Theorem.

LITERATURE CITED

1. F. P. Lockett, "The Fitting class $\mathfrak{F}^*$," Math. Z., <u>137</u>, No. 2, 131-136 (1974).
2. R. A. Bryce and J. A. Cossey, "A problem in the theory of Fitting classes," Math. Z., <u>141</u>, No. 2, 99-110 (1975).
3. J. C. Beidleman and P. Hauck, "Über Fittingklassen und die Lockett-Vermutung," Math. Z., <u>167</u>, No. 2, 161-167 (1979).
4. T. R. Berger and J. Cossey, "An example in the theory of normal Fitting classes," Math. Z., <u>154</u>, No. 3, 287-293 (1977).
5. N. T. Vorob'ev, "Local Fitting classes," Transactions of the 17th All-Union Algebra Conference. Part 2. Minsk, September 14-17, 1983. Inst. Mat. AN BSSR, pp. 42-43.
6. L. A. Shemetkov, Formations of Finite Groups [in Russian], Moscow, Nauka (1978).
7. The Kourovka Notebook (Unsolved Questions in Group Theory) [in Russian], Mat. Inst. AN SSSR (Sib. Branch), Novosibirsk (1982).
8. L. A. Shemetkov, "Subgroups of p-solvable groups," in: Finite Groups [in Russian], Minsk, Nauka i Tekhnika (1975), pp. 207-212.
9. W. Gaschütz, Selected Topics in the Theory of Soluble Groups," Canberra, Austral. Nat. Univ. (1969).
10. P. Hauck, "Eine Bemerkung zur kleisten Fittingklasse," J. Algebra, <u>53</u>, No. 3, 395-401 (1979).
11. N. T. Vorob'ev, "Maximal screens of local formations," Algebra Logika, <u>18</u>, No. 2, 137-161 (1979).
12. N. T. Vorob'ev, "On a criterion for formation products to be local," Mat. Zametki, <u>34</u>, No. 2, 165-170 (1983).
13. H. Laue, "Über nichtauflösbare normale Fittingklassen," J. Algebra, <u>45</u>, No. 2, 274-283 (1977).
14. P. Hauck, "On products of Fitting classes," J. London Math. Soc., <u>20</u>, No. 2, 423-434 (1979).
15. R. A. Bryce and J. Cossey, "Subgroup closed Fitting classes are formations," Math. Proc. Camb. Phil. Soc., <u>91</u>, No. 2, 225-258 (1982).