COROLLARY 3. The quasivariety generated by the singly defined, torsion-free groups cannot be defined by a system of quasiidentities in a finite number of variables.

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MAXIMAL SCREENS OF LOCAL FORMATIONS

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One of the main problems of the theory of formations, from its conception to the present, is the determination and investigation of new formations. Of particular importance for progress on this question is the choice and description of a method of constructing formations. The first steps in this direction were taken by Gaschütz in 1963 in the first paper on the theory of formations [12], in which he gave methods for constructing certain well-known formations via a description of their local assignments. Then, beginning in 1969, there appeared several additional papers [13-16] devoted to this problem, most of which were due to K. Doerk. In this regard, by a local assignment f of a formation $\mathfrak{F}$ was always meant a formation assigning to each prime p a formation $f(p)$ such that $\mathfrak{F} = \langle f \rangle$, where $\langle f \rangle$ is the class of all groups possessing f-central chief series (see, e.g., [11]).

In 1974 Shemetkov [1] introduced the concept of a screen and suggested a classification of screens. Using the concept of a screen, he formulated the general problem of constructing and investigating formations in the following way.

If $\mathfrak{F}$ is a local formation, the problem consists in describing those screens f for which $\mathfrak{F} = \langle f \rangle$. If the given formation $\mathfrak{F}$ is not local, then it is natural to pose the analogous question for the smallest local formation $l\,form\,\mathfrak{F}$ containing the given one.

An important role in the solution of this kind of question is played by the maximal local screens of formations. The main purpose of the present paper is to investigate formations of finite groups by means of maximal local screens.

In Sec. 1 we construct five new types of formations with the aid of two formations (these five being the formation products of the i-th kind: $\mathfrak{F} *_i \mathfrak{H}$, $1 \leq i \leq 5$). In special cases, formation products have been

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studied earlier by various authors. In the class of solvable groups, Carter [26] and Huppert [11] studied the classes $\mathfrak{N} *_3 \mathfrak{N}$ and $\mathfrak{N} *_1 \mathfrak{N}$, Doerk [20] and D'Arcy [25] the classes $\mathfrak{F} *_1 \mathfrak{H}$ and $\mathfrak{F} *_2 \mathfrak{H}$, and Doerk and Hawkes [14] and Doerk [13-16] the classes $\mathfrak{F} *_3 \mathfrak{F}$, $\mathfrak{F} *_4 \mathfrak{F}$, and $\mathfrak{T} *_5 \mathfrak{F}$.

In Sec. 2 formation products are used to obtain an explicit description of the maximal screens of local formations. Note that the problem of determining the maximal screens of formations in special cases was posed by Wright [17] and Schmid [18] and in general form by Shemetkov [5, 6].

In Sec. 3 we describe the maximal screens of local formations generated by formation products, and in Sec. 4 we consider criteria for localness of formation products.

Throughout this paper we will carry out all investigations in some nonempty class of finite groups $\mathfrak{U}$ that is closed under the operators S, Q and $\mathrm{Ext}_{\mathfrak{U}}$. Therefore, by a group we will always mean a group in $\mathfrak{U}$, and all considered classes of groups, in particular, the values of screens on nonidentity groups, are sublcasses of $\mathfrak{U}$.

A local screen f is called:

1) complete if $\mathfrak{N}_p f(p) = f(p)$ for each prime p;

2) S -closed if the formation $f(p)$ is S-closed for each prime p.

A subgroup H of a group G is called a $\mathcal{DM}$ -subgroup [23] if H either covers or avoids each chief factor of G.

All other definitions and notation used in this paper can be found in L. A. Shemetkov's monograph [5].

The main results of the present paper were published without proof in [7-10].

1. Formation Products

In this section we introduce and study the classes $\mathfrak{F} *_i \mathfrak{H}$ $(1 \le i \le 5)$, which we call formation products of the i-th kind.

Definition 1.1. Suppose $\mathfrak{F}$ is a local formation and $\mathfrak{H}$ an arbitrary formation. We denote by $\mathfrak{F} *_1 \mathfrak{H}$ the class of all groups whose $\mathfrak{F}$-normalizers belong to $\mathfrak{H}$, and by $\mathfrak{F} *_2 \mathfrak{H}$ the class of all groups whose $\mathfrak{F}$-projectors belong to $\mathfrak{H}$.

If $\mathfrak{H} = \varnothing$, then we put $\mathfrak{F} *_i \mathfrak{H} = \varnothing$, $i = 1, 2$.

THEOREM 1.1. Suppose $\mathfrak{F}$ is a local formation and $\mathfrak{H}$ a formation. If the $\mathfrak{F}$-coradical of each group in $\mathfrak{U}$ is $\pi(\mathfrak{F})$-solvable, then the class $\mathfrak{F} *_i \mathfrak{H}$ $(i = 1, 2)$ is a formation.

The proof of the theorem is easily obtained from the fact that, according to Theorems 15.7 and 21.4 of [5], all $\mathfrak{F}$-projectors and $\mathfrak{F}$-normalizers of groups with $\pi(\mathfrak{F})$-solvable $\mathfrak{F}$-coradical are conjugate.

Definition 1.2. Suppose $\mathfrak{F}$, $\mathfrak{H}$ are local formations with maximal inner local screens f, h respectively. We construct classes $\mathfrak{F} *_i \mathfrak{H}$ $(i = 3, 4, 5)$ as follows:

1) $G \in \mathfrak{F} *_3 \mathfrak{H}$ if and only if each f^*-central chief factor of G is $\mathfrak{H}$-central, where f^* is the local formation such that $f^*(p) = \mathfrak{F} *_2 f(p)$ for each prime p;

2) $G \in \mathfrak{F} *_4 \mathfrak{H}$ if and only if an $\mathfrak{F}$-projector of G is contained in some $\mathfrak{H}$-normalizer of G;

3) $G \in \mathfrak{F} *_5 \mathfrak{H}$ if and only if a $\mathfrak{H}$-projector of G is a $\mathcal{DM}$-subgroup of G and belongs to $f(p)$ for each prime p.

If $f(p) = \varnothing$ for some prime p, then $\mathfrak{F} *_5 \mathfrak{H} = \varnothing$.

THEOREM 1.2. A formation product of the third kind is a formation.

Proof. Suppose $\mathfrak{F}$ and $\mathfrak{H}$ are local formations and $\mathfrak{F} *_3 \mathfrak{H}$ is the formation product of the third kind. Suppose $K_i \triangleleft G$, $i=1,2$. Obviously, $G \in \mathfrak{F} *_3 \mathfrak{H}$ implies $G/K_i \in \mathfrak{F} *_3 \mathfrak{H}$. Assume that $G/K_i \in \mathfrak{F} *_3 \mathfrak{H}$, $i=1,2$. It is easy to see that each chief factor of the group $G/K_1 \cap K_2$ is G-isomorphic either to some chief factor of G/K_1 or to some chief factor of G/K_2. Therefore, $G/K_1 \cap K_2 \in \mathfrak{F} *_3 \mathfrak{H}$.

The theorem is proved.

It follows from Theorem 21.8 and assertion 21.1.1 of [5] that if the $\mathfrak{F}$-coradical of a group G is $\pi(\mathfrak{F})$-solvable, then an $\mathfrak{F}$-projector of G covers each $\mathfrak{F}$-central chief factor of G.

LEMMA 1.1. Suppose $\mathfrak{X}$ is a formation, $\mathfrak{F}$ is a subformation, and the $\mathfrak{F}$-coradical of each group in $\mathfrak{X}$ is $\pi(\mathfrak{F})$-solvable. If φ is an arbitrary local $\mathfrak{X}$-screen, f is a maximal inner local screen of $\mathfrak{F}$, and $f(p) = \mathfrak{N}_p \varphi(p) \cap \mathfrak{F}$ for each prime p, then φ is a local $\mathfrak{X}$-screen of $\mathfrak{F}$.

Proof. It is easy to see that $\mathfrak{F} \subseteq \langle\varphi\rangle$. Let us prove the reverse inclusion. Assume it is false. Choose in the class $\langle\varphi\rangle \setminus \mathfrak{F}$ a group G of smallest order. Then G has a unique minimal normal subgroup K, which coincides with $G^{\mathfrak{F}}$. If K is non-Abelian, then $C_G(K) = 1$. Consequently, $G \in \varphi(K) \subseteq \mathfrak{X}$ and therefore its order is not divisible by numbers in $\pi(\mathfrak{F})$. But then $f(K) = \emptyset$, which implies $\varphi(K) = \emptyset$. Contradiction. It remains to assume that K is an Abelian p-group for some $p \in \pi(\mathfrak{F})$. Then it is easy to see that $G/C_G(K) \in \varphi(p) \cap \mathfrak{F} \subseteq f(p)$. Consequently, $G \in \mathfrak{F}$, which is impossible.

The lemma is proved.

LEMMA 1.2. Suppose each group in $\mathfrak{U}$ has a $\pi(\mathfrak{F})$-solvable $\mathfrak{F}$-coradical, where $\mathfrak{F}$ is a formation with maximal inner local screen f. If f^* is the local screen such that $f^*(p) = \mathfrak{F} *_2 f(p)$ for each prime p, then:

1) f^* is a complete local screen of $\mathfrak{F}$;

2) if φ is either an inner or an $\mathfrak{S}$-closed local screen of $\mathfrak{F}$, then $\varphi \leqslant f^*$.

Proof. Suppose G is a group in $\mathfrak{N}_p f^*(p)$, where p is a prime, and F is an $\mathfrak{F}$-projector of G. Then $F/O_p(F) \simeq F O_p(G)/O_p(G) \in f(p)$. Since, by Theorem 3.3 of [5], the screen f is complete, it follows that $F \in f(p)$. Consequently, $G \in f^*(p)$, and f^* is a complete local screen. That it is a screen of $\mathfrak{F}$ now follows directly from Lemma 1.1.

Assertion 2) in the case where φ is an inner local screen of $\mathfrak{F}$ is trivial. Suppose φ is an $\mathfrak{S}$-closed local screen of $\mathfrak{F}$. Suppose also that $G \in \varphi(p)$, where p is a prime, and F is an $\mathfrak{F}$-projector of G. Then $F \in \varphi(p) \cap \mathfrak{F}$ and, by Theorem 3.3 of [5], $F \in f(p)$. Consequently, $G \in f^*(p)$.

The lemma is proved.

LEMMA 1.3. Suppose each group of $\mathfrak{U}$ has a $\pi(\mathfrak{F})$-solvable $\mathfrak{F}$-coradical, where $\mathfrak{F}$ is a formation with maximal inner local screen f. Let f^* be the local screen such that $f^*(p) = \mathfrak{F} *_2 f(p)$ for each prime p, and let F be an $\mathfrak{F}$-projector of G. Then:

1) F covers each f^*-central chief factor of G;

2) each chief factor of G covering the subgroup F is f^*-central.

Proof. Let us prove the first assertion. Suppose G is a counterexample of least order and K is a minimal normal subgroup of G. It is easy to show by induction that F covers each f^*-central chief factor of G lying above K. Therefore, to prove 1) it suffices to show that F covers K if K is f^*-central in G.

Obviously, $G^{\mathfrak{F}} \neq 1$. Assume that K is not contained in $G^{\mathfrak{F}}$. Then $KG^{\mathfrak{F}}/G^{\mathfrak{F}} \simeq K$, hence K is $\mathfrak{F}$-central in G. Therefore, F covers K, which is impossible. Suppose K is contained in $G^{\mathfrak{F}}$. Since $G^{\mathfrak{F}}$ is $\pi(\mathfrak{F})$-solvable, it follows that $|K|$ either is not divisible by the numbers in $\pi(\mathfrak{F})$ or is a power of a prime p in $\pi(\mathfrak{F})$. In the first case, by Lemma 1.2, $f^*(K) = \emptyset$, which is impossible. There remains the second case: K is a p-group, $p \in \pi(\mathfrak{F})$. Then, obviously, $FK/C_{FK}(K) \simeq F/C_F(K) \in f(p)$. Consequently, F covers each FK-chief factor of K, hence F covers K. Contradiction.

Let us prove the second assertion of the lemma. Suppose G is a group of least order for which assertion 2) does not hold. Let K be a minimal normal subgroup of G. It is easy to see that to prove 2) it suffices to show that K is f^*-central in G if F covers K. By Lemma 1.2, $G^{\mathfrak{F}} \neq 1$. If K is not contained in $G^{\mathfrak{F}}$, then K is $\mathfrak{F}$-central in G and, by Lemma 1.2, f^*-central in G, which is impossible. Therefore, K is a p-group for some prime p in $\pi(\mathfrak{F})$. By Theorem 3.3 of [5], f is a complete local screen, hence $F/O_{p'}(F) \in f(p)$, and therefore $FC_G(K)/C_G(K) \simeq F/C_F(K) \in f(p)$. Consequently, $G/C_G(K) \in f^*(p)$.

The lemma is proved.

THEOREM 1.3. If each group in $\mathfrak{U}$ has a $\pi(\mathfrak{F})$-solvable $\mathfrak{F}$-coradical, where $\mathfrak{F}$ is a formation with maximal inner local screen f, then $\mathfrak{F} *_3 \mathfrak{F}$ is the formation of all groups in which an $\mathfrak{F}$-projector covers only the $\mathfrak{F}$-central chief factors.

The proof follows directly from Theorem 1.2 and Lemma 1.3.

Note that in the case where $\mathfrak{U} = \mathfrak{S}$ the formation $\mathfrak{F} *_3 \mathfrak{F}$ was studied by Doerk [13] and by Beidleman and Makan [21].

LEMMA 1.4. Suppose G is a group and K_1, K_2 are arbitrary normal subgroups of G. Then:

1) if F is an $\mathfrak{F}$-projector of G, then $FK_1 \cap FK_2 = F(K_1 \cap K_2)$;

2) if H is an $\mathfrak{F}$-normalizer of G and $G^{\mathfrak{F}}$ is $\pi(\mathfrak{F})$-solvable, then $HK_1 \cap HK_2 = H(K_1 \cap K_2)$.

Proof. The first assertion of the lemma follows from Theorem 2.1 and Lemma 2.5 of [22]. Let us prove the second. Obviously, $H(K_1 \cap K_2) \subseteq HK_1 \cap HK_2$. Let us prove the reverse inclusion. Assume it is false. Suppose G is a counterexample of least order. If either $K_1 \cap K_2 \neq 1$ or one of the subgroups K_1, K_2 is 1, then, obviously, $HK_1 \cap HK_2 = H(K_1 \cap K_2)$. Therefore, $K_1 \cap K_2 = 1$ and $K_1 \neq 1$, $K_2 \neq 1$. Let N be a minimal normal subgroup of G contained in K_1. Then, by induction, $HK_1 \cap HNK_2 = HK_2$. Consequently, $HK_1 \cap HK_2 = HN \cap HK_2$, hence $N = K_1$ Analogously, it is easy to see that K_2 is a minimal normal subgroup of G. In view of assertion 21.11 of [5], it follows that K_1, K_2 are $\mathfrak{F}$-eccentric chief factors of G. Since K_1K_2/K_1 is an $\mathfrak{F}$-eccentric chief factor of G, it follows easily from Theorem 21.1 of [5] that $H \cap K_1K_2 = 1$. Suppose x is an arbitrary element of the group $HK_1 \cap HK_2$. Then $x = h_1k_1 = h_2k_2$, where $h_1, h_2 \in H$, $k_1 \in K_1$, $k_2 \in K_2$. Consequently, $h_2^{-1}h_1 = k_2k_1^{-1}$ is an element of $H \cap K_1K_2 = 1$. Therefore, $x \in H$. Thus, $HK_1 \cap HK_2 \subseteq H$, which is impossible.

The lemma is proved.

THEOREM 1.4. If each group in $\mathfrak{U}$ has a $\pi(\mathfrak{F}_i)$-solvable $\mathfrak{F}_i$-coradical, where $\mathfrak{F}_i$ is a local formation $(i=1,2)$, then $\mathfrak{F}_1 *_4 \mathfrak{F}_2$ is a formation.

The proof of the theorem is obtained by using Lemma 1.4.

From Theorem 1.4 and Theorem 21.8 of [5] we have

COROLLARY 1.4.1. If each group in $\mathfrak{M}$ has a $\pi(\mathfrak{F})$ -solvable $\mathfrak{F}$ -coradical, where $\mathfrak{F}$ is a local formation, then $\mathfrak{F} *_4 \mathfrak{F}$ coincides with the class of all groups in which an $\mathfrak{F}$ -projector coincides with an $\mathfrak{F}$ -normalizer.

Note that the formation $\mathfrak{F} *_4 \mathfrak{F}$ in the case where $\mathfrak{M} = \mathfrak{S}$ and $\mathfrak{F} \supseteq \mathfrak{N}$ was studied in [13-15].

THEOREM 1.5. Suppose $\mathfrak{F}, \mathfrak{H}$ are local formations. If each group $\mathfrak{M}$ has a $\pi(\mathfrak{H})$ -solvable $\mathfrak{H}$ -coradical, then $\mathfrak{F} *_5 \mathfrak{H}$ is a formation.

Proof. Suppose K is a normal subgroup of G. Obviously, $G \in \mathfrak{F} *_5 \mathfrak{H}$ implies $G/K \in \mathfrak{F} *_5 \mathfrak{H}$. Assume that K_1, K_2 are distinct minimal normal subgroups of G such that $G/K_i \in \mathfrak{F} *_5 \mathfrak{H}$, $i=1,2$. We will prove that $G \in \mathfrak{F} *_5 \mathfrak{H}$. Obviously, the $\mathfrak{H}$ -projectors of G belong to $f(p)$ for each prime p. Suppose H/K is an arbitrary chief factor of G lying above K_1, and F is some $\mathfrak{H}$ -projector of G. Since $G/K_1 \in \mathfrak{F} *_5 \mathfrak{H}$, it is obvious that F either covers or avoids each chief factor of G lying above K_1. Also, $G/K_2 \in \mathfrak{F} *_5 \mathfrak{H}$ implies that FK_2/K_2 either covers or avoids K_1K_2/K_2. If FK_2/K_2 avoids K_1K_2/K_2, then it is easy to see that $F \cap K_1 = 1$; hence F avoids K_1. If FK_2/K_2 covers K_1K_2/K_2, then $FK_2 \supseteq K_1K_2$. Therefore, by Theorem 2.1 of [22] $K_1K_2 = K_2(F \cap K_1)$. Consequently, $F \cap K_1 = K_1$, hence F covers K_1. Thus we have shown that F either covers or avoids each chief factor of G in a chief series passing through K_1.

Suppose R/S is an arbitrary chief factor of G and h^* is the local screen such that $h^*(p) = \mathfrak{H} *_2 h(p)$ for each prime p. If R/S is an h^* -central chief factor, then, by Lemma 1.3, F covers R/S. Thus, we may assume that R/S is an h^* -eccentric chief factor of G. We consider two cases.

1. The subgroup K_1 covers R/S. Then, obviously, $R/S \simeq K_1$. Therefore, by Lemma 1.3, $F \cap K_1 = 1$. By Theorem 2.1 of [22], $F \cap R \subseteq F \cap K_1 S = F \cap S$, hence F avoids S.

2. The subgroup K_1 avoids R/S.

In this case $R/S \simeq K_1R/K_1S$, hence it follows from Lemma 1.3 that F avoids K_1R/K_1S. By Theorem 2.1 of [22], $F \cap R \subseteq (F \cap K_1)(F \cap S)$. Therefore, applying the same theorem again, we obtain $F \cap R = (F \cap S)(F \cap K_1 \cap R) \subseteq S$. Consequently, F avoids R/S. Thus, F is a $\mathfrak{DM}$ -subgroup of G.

The theorem is proved.

2. Maximal Screens of Local Formations

In [7] we announced the following theorem, in which are used formation products of the first kind.

THEOREM 2.1. Suppose $\mathfrak{F} \subseteq \mathfrak{X}$ are certain formations, $\mathfrak{F}$ is local, and each group in $\mathfrak{X}$ has a $\pi(\mathfrak{F})$-solvable $\mathfrak{F}$ -coradical. Then:

1) $\mathfrak{F}$ has a unique maximal local $\mathfrak{X}$ -screen f;

2) if φ is a maximal inner local screen of $\mathfrak{F}$, then $f(p) = (\mathfrak{F} *_1 \varphi(p)) \cap \mathfrak{X}$ for each prime p.

This theorem is discussed in detail in [5, p. 22], hence we omit the proof here.

We mention only that Theorem 2.1 with $\mathfrak{M} = \mathfrak{G}$ and $\mathfrak{X} = \mathfrak{F}$ includes the results of Carter and Hawkes [19] and Schmid [18], and with $\mathfrak{M} = \mathfrak{X} = \mathfrak{S}$ and $\mathfrak{F} \supseteq \mathfrak{M}$ the result of Doerk [20].

Definition 2.1. Suppose $\mathfrak{F}$ is a formation with local screen f and $\mathfrak{X}$ is some class of groups with a $\pi(\mathfrak{F})$ -solvable $\mathfrak{F}$ -coradical. The screen f of the formation $\mathfrak{F}$ is called $\mathfrak{X}$ -monotone if, for each group $G \in \mathfrak{X}$ and $\mathfrak{F}$ -projector F of this group, $F \subseteq K \subseteq L \subseteq G$ always implies that $K^{f(p)} \subseteq L^{f(p)}$ for all primes $p \in \pi(\mathfrak{F})$.

THEOREM 2.2. Suppose $\mathfrak{F} \subseteq \mathfrak{X}$ are certain formations, $\mathfrak{F}$ is local, $\mathfrak{X}$ is S -closed, and each group in $\mathfrak{X}$ has a $\pi(\mathfrak{F})$ -solvable $\mathfrak{F}$ -coradical. Then:

1) $\mathfrak{F}$ has a unique maximal $\mathfrak{X}$ -monotone local $\mathfrak{X}$ -screen f ;

2) if ψ is a maximal inner local screen of $\mathfrak{F}$, then $f(p) = (\mathfrak{F} *_2 \psi(p)) \cap \mathfrak{X}$ for any prime p .

Proof. By Theorem 3.3 of [5], the formation $\mathfrak{F}$ has a unique maximal inner local screen ψ . Let f be the local screen such that $f(p) = (\mathfrak{F} *_2 \psi(p)) \cap \mathfrak{X}$ for any prime p (obviously, $(\mathfrak{F} *_2 \psi(p)) \cap \mathfrak{X}$ is a formation). We will prove that f is an $\mathfrak{X}$ -monotone local screen. Suppose the group G belongs to $\mathfrak{X}$ and F is an -projector of G . Take subgroups K and L of G, such that $F \subseteq K \subseteq L \subseteq G$. Let p be a prime belonging to $\pi(\mathfrak{F})$. Obviously, $\psi \leq f$. Therefore, $f(p) \neq \emptyset$. Since F is an $\mathfrak{F}$ -projector of K belonging to $\mathfrak{X}$, it follows that $F^{\psi(p)} \subseteq K^{f(p)}$. Assume that N is a normal subgroup of G such that $F^{\psi(p)} \subseteq N$ and $N \subset K^{f(p)}$. Then the $\mathfrak{F}$ -projector FN/N of the group K/N belongs to $\psi(p)$, hence $K/N \in f(p)$. Consequently, $K^{f(p)}$ is the smallest normal subgroup of K containing $F^{\psi(p)}$. Analogously, $L^{f(p)}$ is the smallest normal subgroups of L containing $F^{\psi(p)}$. Since $L^{f(p)} \cap K$ is normal in K , it follows that $K^{f(p)} \subseteq L^{f(p)}$. Thus, f is an $\mathfrak{X}$ -monotone local $\mathfrak{X}$ -screen. Thus $\mathfrak{F} = \langle f \rangle$, follows easily from Lemma 1.1.

Suppose f_1 is an arbitrary $\mathfrak{X}$ -monotone local $\mathfrak{X}$ -screen of $\mathfrak{F}$. Let φ be the inner screen of $\mathfrak{F}$ such that $\varphi(p) = f_1(p) \cap \mathfrak{F}$ for each prime p . Then $\varphi \leq \psi$. Assume that the group G belongs to $f_1(p)$, where $p \in \pi(\mathfrak{F})$. Let F be an $\mathfrak{F}$ -projector of G . Then $F^{f_1(p)} \subseteq G^{f_1(p)}$, hence $F \in \psi(p)$. But then $G \in f(p)$. Thus, $f_1 \leq f$.

The theorem is proved.

COROLLARY 2.2.1. Suppose $\mathfrak{F} \subseteq \mathfrak{X}$ are certain formations, $\mathfrak{F}$ is local, $\mathfrak{X}$ is S -closed, and each group in $\mathfrak{X}$ has a nilpotent $\mathfrak{F}$ -coradical. If ψ is a maximal inner local screen of $\mathfrak{F}$, then the local screen f such that $f(p) = (\mathfrak{F} *_1 \psi(p)) \cap \mathfrak{X}$ for each prime p is a maximal $\mathfrak{X}$ -monotone local $\mathfrak{X}$ - screen of $\mathfrak{F}$.

The proof of this assertion follows directly from the previous theorem and Theorem 21.5 of [5].

3. Maximal Screens of Induced Local Formations

LEMMA 3.1. Suppose f_1, f_2 are maximal inner local screens of the formations $\mathfrak{F}_1, \mathfrak{F}_2$, respectively. Then $\mathfrak{F}_1$ is a subformation of $\mathfrak{F}_2$ if and only if $f_1 \leq f_2$.

We omit the proof of the lemma, since it is given in [5, p. 65].

It is easy to establish

LEMMA 3.2. If $\mathfrak{F}$ is an arbitrary nonempty formation, then the local screen f such that $f(p) = \mathfrak{N}_p \mathfrak{F}$ for each prime p is a maximal inner local screen of the formation $\mathfrak{N}\mathfrak{F}$.

LEMMA 3.3. Suppose all minimal normal subgroups of a group G are solvable. If G has at most two minimal normal subgroups and $O_p(G) = 1$ for some prime p , then G has a faithful irreducible representation over a finite field of characteristic p .

Proof. If G is simple, the lemma is trivial. Suppose M is the product of all minimal normal subgroups of G . Obviously, $M = \langle m \rangle \times K$, where $K_G = 1$ and $n = |\langle m \rangle|$ is either a prime or the product of two distinct primes, and $(n, p) = 1$. Consequently, in some finite field of characteristic p there exists a primitive n-th root of unity ε . Consider the mapping $\varphi : m^{\alpha} k \longrightarrow \varepsilon^{\alpha}$, where $k \in K$. Obviously, φ is a one-dimensional representation of M with kernel K. Let φ^G be the representation of G induced by the representation φ of M. Look at the irreducible constituent $(\tau(g))$ in the upper left corner of the matrix $(\varphi^G(g))$, where $g \in G$. It is easy to see that $Ker\,\tau \cap M = K_G = 1$. Therefore, $Ker\,\tau = 1$ and τ is the desired representation of G.

The lemma is proved.

THEOREM 3.1. Suppose $\mathfrak{U} = \Gamma$ and $\mathfrak{F} \subseteq \mathfrak{H}$ are certain formations. The local screen f such that $f(p) = \mathfrak{N}_p(\mathfrak{F} *_1 \mathfrak{H})$ for each prime p is a maximal inner local screen of the formation $l form(\mathfrak{F} *_1 \mathfrak{H})$.

Proof. Suppose f is a maximal inner local screen of the formation $l form(\mathfrak{F} *_1 \mathfrak{H})$. Since $\mathfrak{F} *_1 \mathfrak{H} \subseteq \mathfrak{N}(\mathfrak{F} *_1 \mathfrak{H})$, it follows that $l form(\mathfrak{F} *_1 \mathfrak{H}) \subseteq \mathfrak{N}(\mathfrak{F} *_1 \mathfrak{H})$. Therefore, by Lemmas 3.1 and 3.2, $f(p) \subseteq \mathfrak{N}_p(\mathfrak{F} *_1 \mathfrak{H})$. Let us prove the reverse inclusion. Assume it is false. Choose in the class $\mathfrak{N}_p(\mathfrak{F} *_1 \mathfrak{H}) \setminus f(p)$ a group G of least order. Then G has a unique minimal normal subgroup K, which coincides with $G^{f(p)}$. Obviously, $O_p(G) = 1$. Therefore, by Lemma 3.3, G is an irreducible group of automorphisms of some p-group N. Let $\Gamma = N \rtimes G$ be the extension of G by N. It is easy to see that G does not belong to $\mathfrak{F}$. Then G is a maximal $\mathfrak{F}$-abnormal subgroup of Γ. Therefore, by Lemma 13.3 of [5], G is $\mathfrak{F}$-critical in Γ. Consequently, an $\mathfrak{F}$-normalizer H of G is an $\mathfrak{F}$-normalizer of Γ. It follows that $\Gamma \in \mathfrak{F} *_1 \mathfrak{H} \subseteq l form(\mathfrak{F} *_1 \mathfrak{H})$. Therefore, $G \simeq \Gamma/N \in f(p)$. Contradiction.

The theorem is proved.

THEOREM 3.2. Suppose each group in $\mathfrak{U}$ has a $\pi(\mathfrak{F})$-solvable $\mathfrak{F}$-coradical, where $\mathfrak{F}$ is a formation with maximal inner local screen ψ. Suppose $\mathfrak{H}$ is a formation such that $\mathfrak{N}_p\mathfrak{H} = \mathfrak{H} \subseteq \mathfrak{F} *_2 \psi(p)$ for all primes $p \in \pi(\mathfrak{F})$. Then the local screen f such that

$$f(p) = \begin{cases} \emptyset, & \text{if } \mathfrak{H} = \emptyset, \\ \mathfrak{N}_p(\mathfrak{F} *_2 \mathfrak{H}) & \text{, if } \mathfrak{H} \neq \emptyset, \end{cases}$$

for each prime p is a maximal inner local screen of the formation $l form(\mathfrak{F} *_2 \mathfrak{H})$.

Proof. By Theorem 3.3 of [5], $l form(\mathfrak{F} *_2 \mathfrak{H})$ has a unique maximal inner local screen f. If $\mathfrak{H} = \emptyset$, then $\mathfrak{F} *_2 \mathfrak{H} = \emptyset$ and therefore $l form(\mathfrak{F} *_2 \mathfrak{H}) = \mathfrak{E}$, where $\mathfrak{E}$ is a formation of identity groups. Consequently, $f(p) = \emptyset$ for all primes p.

If $\mathfrak{H} \neq \emptyset$, then, in view of Lemmas 3.1 and 3.2, $f(p) \subseteq \mathfrak{N}_p(\mathfrak{F} *_2 \mathfrak{H})$ for all primes p. Suppose G is a group of least order in the class $\mathfrak{N}_p(\mathfrak{F} *_2 \mathfrak{H}) \setminus f(p)$ and F is an $\mathfrak{F}$-projector of G. Consider the regular wreath product, $\Gamma = P \wr G$, where $|P| = p$. Then $\Gamma = N \rtimes G$, where N is an elementary Abelian p-group. Obviously, $N = F_p(\Gamma)$. If $\psi(p) = \emptyset$, then F is an $\mathfrak{F}$-projector of FN, hence an $\mathfrak{F}$-projector of Γ. Consequently, $\Gamma \in \mathfrak{F} *_2 \mathfrak{H} \subseteq l form(\mathfrak{F} *_2 \mathfrak{H})$ and $G \simeq \Gamma/F(\Gamma) \in f(p)$. Contradiction. Suppose $\psi(p) \neq \emptyset$. Then it follows from Theorem 15.7 of [5] and Lemma 2 of [23] that FN is an $\mathfrak{F}$-projector of Γ. Since $FN \in \mathfrak{N}_p\mathfrak{H} = \mathfrak{H}$, it follows that $\Gamma \in \mathfrak{F} *_2 \mathfrak{H}$. Therefore, $G \simeq \Gamma/F_{p'}(\Gamma) \in f(p)$. Contradiction.

The theorem is proved.

LEMMA 3.4. Suppose $\mathfrak{F}_1, \mathfrak{F}_2$ are formations with maximal inner local screens f_1, f_2. Then the local group function φ such that

$$\varphi(p) = ((\mathfrak{U} \setminus (\mathfrak{F}_1 *_2 f_1(p)) \cap (\mathfrak{F}_1 *_3 \mathfrak{F}_2)) \cup f_2(p)$$

for each prime p is a local group function of the formation $\mathfrak{F}_1 *_3 \mathfrak{F}_2$.

Proof. We will show that $\langle\varphi\rangle = \mathfrak{F}_1 *_3 \mathfrak{F}_2$ for the group function φ indicated in the lemma.

Suppose G is a group of least order in the class $(\mathfrak{F}_1 *_3 \mathfrak{F}_2) \setminus \langle\varphi\rangle$. Then G has a unique minimal normal subgroup K, which coincides with $G^{\langle\varphi\rangle}$. Obviously, K is f_1^* -eccentric, where f_1^* is the local group function such that $f_1^*(p) = \mathfrak{F} *_2 f_1(p)$ for each prime p . But then,

$$G/C_G(K) \in (\mathfrak{U} \setminus f_1^*(p)) \cap (\mathfrak{F}_1 *_3 \mathfrak{F}_2) \subseteq \varphi(p).$$

Consequently, $G \in \langle\varphi\rangle$. Contradiction.

It remains to show that $\mathfrak{F}_1 *_3 \mathfrak{F}_2 \supseteq \langle\varphi\rangle$. This follows by induction, inasmuch as $\varphi(p) \cap f_1^*(p) \subseteq f_2(p)$ for each prime p .

The lemma is proved.

LEMMA 3.5. Suppose each group in $\mathfrak{U}$ has a solvable $f_1(p)$ -coradical for all primes $p \in \pi(\mathfrak{F}_1)$, where $\mathfrak{F}_1$ is a formation with maximal inner local screen f_1 . Suppose $\mathfrak{F}_2$ is a formation with maximal inner local screen f_2 . If $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$, and f is a maximal inner local screen of the formation $l\,form\,(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$ then:

1) $\varphi \leq f$, where φ is the local group function of the formation $\mathfrak{F}_1 *_3 \mathfrak{F}_2$, such that

$$\varphi(p) = ((\mathfrak{U} \setminus (\mathfrak{F}_1 *_2 f_1(p)) \cap (\mathfrak{F}_1 *_3 \mathfrak{F}_2)) \cup f_2(p)$$

for each prime p ;

2) $f = \psi$, where ψ is the local screen such that $\psi(p) = \mathfrak{N}_p\,form\,\varphi(p)$ for each prime p and φ is the group function in 1).

Proof. Suppose f_1^* is the local screen such that $f_1^*(p) = \mathfrak{F}_1 *_2 f_1(p)$ for each prime p and φ is the local group function such that

$$\varphi(p) = ((\mathfrak{U} \setminus f_1^*(p)) \cap (\mathfrak{F}_1 *_3 \mathfrak{F}_2)) \cup f_2(p)$$

for each prime p . Then, by Lemma 3.4, φ is an inner local group function of the formation $\mathfrak{F}_1 *_3 \mathfrak{F}_2$.

Let us prove the first assertion of the lemma. Assume it is false. Then there exists a prime p such that $\varphi(p)$ is not contained in $f(p)$, where f is a maximal inner screen of the formation $l\,form\,(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. Choose in the class $\varphi(p) \setminus f(p)$ a group G of least order.

If $f_1(p) = \emptyset$, then $\varphi(p) = \mathfrak{F}_1 *_3 \mathfrak{F}_2$. Obviously, $O_p(G) = 1$. Consider the regular wreath product $\Gamma = P \wr G$, where $|P| = p$. Then $\Gamma = N \leftthreetimes G$, where N is an elementary Abelian p -group. Obviously, $N = O_p(\Gamma) = F_p(\Gamma)$. Since $\Gamma/O_p(\Gamma) \in \varphi(p)$, it follows that $\Gamma \in l\,form\,(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. Thus, $G \simeq \Gamma/F_p(\Gamma) \in f(p)$. Contradiction.

Assume that $f_1(p) \neq \emptyset$. If $G \in f_1^*(p)$, then $G \in \varphi(p) \cap f_1^*(p) \subseteq f_2(p)$. But $\mathfrak{F}_2 \subseteq l\,form\,(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. Therefore, by Lemma 3.1, $f_2(p) \subseteq f(p)$. Consequently, $G \in f(p)$. Contradiction. Thus, we may assume that G does not belong to $f_1^*(p)$.

If has two distinct minimal normal subgroups K_1 and K_2 such that $G/K_i \in \varphi(p)$, $i = 1, 2$, then $G/K_i \in f(p)$ and therefore $G \in f(p)$, which is impossible. If G/K_i does not belong to $\varphi(p)$, $i = 1, 2$, then $G/K_i \in f_1^*(p)$. Consequently, $G \in f_1^*(p)$. Contradiction.

Assume that G has at most two minimal normal subgroups. If G has exactly two distinct minimal normal subgroups K_1 and K_2, where $G/K_1 \in \varphi(p)$ and G/K_2 does not belong to $\varphi(p)$, then $G/K_1 \in f_1(p)$ and $G/K_2 \in f_1^*(p)$. Obviously, K_1 and K_2 are subgroups of the $f_1(p)$-coradical of G. Consequently, K_1 and K_2 are solvable and $O_p(G)=1$. Therefore, by Lemma 3.3, G is an irreducible group of automorphisms of some p-group N. Let $\Gamma = N \leftthreetimes G$ be the extension of G by N. Since $\Gamma/N \simeq G \in \mathfrak{F}_1 *_3 \mathfrak{F}_2$ and N is φ-central in Γ, it follows from Lemma 3.4 that $\Gamma \in \mathfrak{F}_1 *_3 \mathfrak{F}_2$. Therefore, $G \simeq \Gamma/N \in f(p)$. Contradiction.

The case where G has a unique minimal normal subgroup is handled analogously.

Let us prove the second assertion of the lemma. Let ψ_1 be the local screen such that $\psi_1(p) = \mathrm{form}\, \varphi(p)$ for each prime p. Since, according to 1), we have $\varphi \leq f$, it follows that $\psi_1 \leq f$. Obviously, $\langle\psi\rangle = \langle\psi_1\rangle$. Consequently, $\langle\psi\rangle \subseteq l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. On the other hand, it follows from the definition of $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$ that $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2) \subseteq \langle\psi\rangle$. Assertion 2) is now obvious, in view of Theorem 3.3 of [5].

The lemma is proved.

THEOREM 3.3. Suppose $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ are local formations and f_1, f_2 are maximal inner local screens of $\mathfrak{F}_1$, $\mathfrak{F}_2$, respectively. If each group in $\mathfrak{U}$ has a solvable $f_1(p)$-coradical for all primes $p \in \pi(\mathfrak{F}_1)$, then the local screen f such that

$$f(p) = \begin{cases} f_2(p), & \text{if } f_1(p) = \mathfrak{F}_1, \\ \mathfrak{N}_p(\mathfrak{F}_1 *_3 \mathfrak{F}_2), & \text{if } f_1(p) \neq \mathfrak{F}_1, \end{cases}$$

for each prime p is a maximal inner local screen of the formation $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$.

Proof. Let $\mathfrak{F}^* = \mathfrak{F}_1 *_3 \mathfrak{F}_2$ and $\mathfrak{F} = l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. By Lemma 3.5, $\mathfrak{F}$ possesses a unique maximal inner local screen f such that $f(p) = \mathfrak{N}_p \mathrm{form}\, \varphi(p)$ for each prime p, where $\varphi(p) = ((\mathfrak{U} \setminus (\mathfrak{F}_1 *_2 f_1(p)) \cap \mathfrak{F}^*) \cup f_2(p)$ for each prime p. Therefore, to prove the theorem it suffices to clarify the structure of $\mathrm{form}\, \varphi(p)$ for all primes p.

Suppose $f_1(p) = \mathfrak{F}_1$, where p is a prime. Then $\varphi(p) = f_2(p)$, hence $\mathrm{form}\, \varphi(p) = f_2(p)$. Since f_2 is a complete screen, $f_1(p) \neq \mathfrak{F}_1$.

Assume that $f(p) = f_2(p)$, where p is a prime. If $f_1(p) = \varnothing$, then $\varphi(p) = \mathfrak{F}^*$, hence $f(p) = \mathfrak{N}_p \mathfrak{F}^*$.

Suppose $f_1(p) \neq \varnothing$. We will prove that $\mathfrak{F}^* = \mathrm{form}\, \varphi(p)$. Obviously, $\mathrm{form}\, \varphi(p) \subseteq \mathfrak{F}^*$. Let us prove the reverse inclusion. Suppose X is an arbitrary group in $\mathfrak{F}^*$. Choose in the class $\mathfrak{F}_1 \setminus f_1(p)$ a group G of least order. Then G has a unique minimal normal subgroup K, which coincides with $G^{f_1(p)}$. Obviously, K is solvable and $O_p(G) = 1$. Therefore, by Lemma 3.3, G is an irreducible group of automorphisms of some p-group N. Let $\Gamma = N \leftthreetimes G$. Since $\Gamma/N \simeq G \in \varphi(p)$, we have $\Gamma \in \mathfrak{F}^*$, by virtue of Lemma 3.4. It is easy to see that $\Gamma \in \varphi(p)$. Let $\Gamma_1 = \Gamma \times X$. Obviously, $\Gamma_1 \in \varphi(p) \subseteq \mathrm{form}\, \varphi(p)$. Therefore, $X \in \mathrm{form}\, \varphi(p)$ and $\mathfrak{F}^* = \mathrm{form}\, \varphi(p)$. It follows that $f(p) = \mathfrak{N}_p \mathfrak{F}^*$.

The theorem is proved.

LEMMA 3.6. If $\mathfrak{F}$ is a nonempty formation, π is some set of primes, and $(\mathfrak{G}_\pi \cap \mathfrak{U})\mathfrak{F} = \mathfrak{F}$, then the local screen f such that

$$f(p)=\begin{cases}\mathfrak{F}, & \text{if } p\in\pi,\\ (\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}, & \text{if } p\in\pi',\end{cases}$$

for each prime p is a maximal inner local screen of the formation $(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}$.

Proof. We will prove that $(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}=\langle f\rangle$. Obviously, $(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}\subseteq\langle f\rangle$. Let us prove that $\langle f\rangle\subseteq(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}$. Assume this is not so. Let G be a group of least order in the class $\langle f\rangle\setminus(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}$. Then G possesses a unique minimal normal subgroup K. Suppose $O_{\pi'}(G/K)=L/K$. Then $G/L\in\mathfrak{F}$ and $L/K\in\mathfrak{G}_{\pi'}\cap\mathfrak{U}$. We will assume that L is a subgroup of least order such that $G/L\in\mathfrak{F}$ and $L/K\in\mathfrak{G}_{\pi'}\cap\mathfrak{U}$.

If K is non-Abelian, then $C_G(K)=1$ and therefore $G\in(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}$, which is impossible. Suppose K is Abelian. If $K\in\mathfrak{G}_{\pi'}\cap\mathfrak{U}$, then $L\in\mathfrak{G}_{\pi'}\cap\mathfrak{U}$ and therefore $G\in(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}$. Contradiction. If $K\in\mathfrak{G}_{\pi}\cap\mathfrak{U}$, then $G/C_G(K)\in\mathfrak{F}$. Consequently, $G/C_G(K)\cap L\in\mathfrak{F}$ and $L\subseteq C_G(K)$, in view of the minimality of L. By the Schur–Zassenhaus theorem, K has a complement N in L. It is easy to see that N is a normal π'-Hall subgroup of L. But then N is a normal subgroup of G. Consequently, $N=1$, hence $G\in(\mathfrak{G}_{\pi}\cap\mathfrak{U})\mathfrak{F}=\mathfrak{F}=(\mathfrak{G}_{\pi'}\cap\mathfrak{U})\mathfrak{F}$. Contradiction. The lemma now follows from Theorem 3.3 of [5].

The lemma is proved.

THEOREM 3.4. Suppose $\mathfrak{F}_1\subseteq\mathfrak{F}_2$ are local formations and f_1, f_2 are maximal inner local screens of $\mathfrak{F}_1$, $\mathfrak{F}_2$, respectively. If $\mathfrak{U}$ is the set of all $\pi(\mathfrak{F}_2)$-solvable groups in $\mathfrak{G}$, then the local screen f such that

$$f(p)=\begin{cases}f_2(p), & \text{if } f_1(p)=\mathfrak{F}_1,\\ \mathfrak{N}_p(\mathfrak{F}_1 *_4 \mathfrak{F}_2) & \text{if } f_1(p)\neq\mathfrak{F}_1,\end{cases}$$

for each prime p is a maximal inner local screen of the formation $l\mathit{form}(\mathfrak{F}_1 *_4 \mathfrak{F}_2)$.

Proof. Let $\mathfrak{F}=l\mathit{form}\,\mathfrak{F}^*$, where $\mathfrak{F}^*=\mathfrak{F}_1 *_4 \mathfrak{F}_2$. By Theorem 3.3 of [5], the formation $\mathfrak{F}$ has a unique maximal inner local screen f.

Suppose $f_1(p)=\mathfrak{F}_1$, where p is a prime. We will prove that $f(p)=f_2(p)$. Obviously, $\mathfrak{F}_2\subseteq\mathfrak{F}$. Therefore, by Lemma 3.1, $f_2\leq f$, hence $f_2(p)\subseteq f(p)$. Suppose G is a group in $\mathfrak{F}^*$. Since $f_1(p)=\mathfrak{F}_1$ it follows from Lemma 1.3 that an $\mathfrak{F}_1$-projector of G covers each chief factor of G whose order is divisible by p. But then an $\mathfrak{F}_2$-normalizer of G covers these same factors. Therefore, by Theorem 21.1.1 of [5], each of them is $\mathfrak{F}_2$-central. Therefore, $G\in(\mathfrak{G}_{p'}\cap\mathfrak{U})f_2(p)$. Thus, $\mathfrak{F}^*\subseteq(\mathfrak{G}_{p'}\cap\mathfrak{U})f_2(p)$, hence $\mathfrak{F}\subseteq(\mathfrak{G}_{p'}\cap\mathfrak{U})f_2(p)$. But then, by Lemmas 3.1 and 3.6, $f(p)\subseteq f_2(p)$.

Assume that $f_1(p)\neq\mathfrak{F}_1$, where p is a prime. We will prove that $f(p)=\mathfrak{N}_p\mathfrak{F}^*$. Since $\mathfrak{F}\subseteq\mathfrak{N}\mathfrak{F}^*$, it follows from Lemmas 3.1 and 3.2 that $f(p)\subseteq\mathfrak{N}_p\mathfrak{F}^*$. Let G be a group of least order in $\mathfrak{N}_p\mathfrak{F}^*\setminus f(p)$. Obviously, $G\in\mathfrak{F}^*$.

Suppose $f_1(p)=\emptyset$. Consider the regular wreath product $\Gamma=P\wr G$, where $|P|=p$. Then $\Gamma=N\rtimes G$, where N is an elementary Abelian p-group. Obviously, $N=F_p(\Gamma)$. Suppose F is an $\mathfrak{F}_1$-projector of G. Then, obviously, F is an $\mathfrak{F}_1$-projector of FN, hence F is an $\mathfrak{F}_1$-projector of Γ. Since $G\in\mathfrak{F}^*$, it follows that $F\subseteq H$, where H is some $\mathfrak{F}_2$-normalizer of G. By Theorem 21.6 of [5], $H=G\cap H^*$, where H^* is some $\mathfrak{F}_2$-normalizer of Γ. Therefore, $\Gamma\in\mathfrak{F}^*\subseteq\mathfrak{F}$, hence $G\simeq\Gamma/F_p(\Gamma)\in f(p)$. Contradiction.

Assume that $f_1(p) \neq \varnothing$. We will first prove that the class $\mathfrak{F}_1 \setminus f_1(p)$ contains a group X such that:

1) X has a unique minimal normal subgroup whose order is a p'-number;

2) $Z(X)$ is a proper subgroup of the $f_1(p)$-coradical of X.

Choose in the class $\mathfrak{F}_1 \setminus f_1(p)$ a group Y of least order. Then Y has a unique minimal normal subgroup K, which coincides with $Y^{f_1(p)}$. Obviously, $O_p(Y) = 1$. Suppose K is a q-group for some prime $q \neq p$ and $Y \in f_1(q)$. Let Q be a q-complement of Y and M a Y-module over a field of q elements induced by an irreducible trivial Q-module over the same field. Then M is a principal indecomposable module and its socle is an irreducible trivial Y-module over the field of q elements. Obviously, M is a faithful module. Let $X = M \leftthreetimes Y$. Since the module M is faithful, $Z(X) \subseteq M$. But the socle of the module M is the unique minimal normal subgroup of X. Therefore, $Z(X) = M$ and requirement 1) for X is satisfied. Since $X^{f_1(p)}$ is not contained in M and $Z(X) \subseteq X^{f_1(p)}$, it follows that $Z(X)$ is a proper subgroup of $X^{f_1(p)}$ and condition 2) for X is satisfied.

Let $X^* = X \times G$. Obviously, the subgroup $F_1 = X \times F$ is an $\mathfrak{F}_1$-projector of X^*. Suppose $F^* = F_1^{f_1(p)} \cap X = 1$. Then, by Lemma 1.1 of [14], $F_1^{f_1(p)} \subseteq Z(X) \times F$. Consequently, $X/Z(X) \simeq X^*/Z(X) \times F \in f_1(p)$, which contradicts the fact that $Z(X)$ is a proper subgroup of $X^{f_1(p)}$. Thus, we may assume that $F^* \neq 1$.

By Lemma 3.3, X possesses a faithful irreducible X-module N over a field of p elements. Therefore, since $F^* \neq 1$, we have $C_N(F^*) = 1$. Suppose R is a regular G-module over the field of p elements. Consider the tensor product $M^* = N \otimes R$ with operator $(n \otimes r)(x, g) = nx \otimes rg$, where $r \in R$, $n \in N$, $x \in X$, $g \in G$. Let $M^*|_X$ be the restriction of the X^*-module M^* to X. Then $M^*|_X \simeq \underbrace{N \oplus \ldots \oplus N}_{|G|}$. Therefore, $C_{M^*}(F^*) = 1$. Let $\Gamma^* = M^* \leftthreetimes (X \times G)$. Then, by Theorem 15.7 of [5] and Lemma 2 of [24], F_1 is an $\mathfrak{F}_1$-projector of the group $F_1 M^*$, hence F_1 is an $\mathfrak{F}_1$-projector of Γ^*. Obviously, $X^* \in \mathfrak{F}_1 *_4 \mathfrak{F}_2$. Therefore, $F_1 \subseteq H_1$, where H_1 is some $\mathfrak{F}_2$-normalizer of X^*. But then $H_1 = X^* \cap H^*$ for some $\mathfrak{F}_2$-normalizer of Γ^*. Therefore, $\Gamma^* \in l\,form(\mathfrak{F}_1 *_4 \mathfrak{F}_2)$. It is easy to see that M^* contains an X^*-submodule over the field of p elements of $\mathcal{F}$ isomorphic to the X^*-module $N \otimes 1_G$, the kernel of which is G, and the restriction $M^*|_G$ of M^* to G is isomorphic to $\underbrace{R \oplus \ldots \oplus R}_{\dim \in N}$. It follows that M^* is a faithful X^*-module over $\mathcal{F}$. Therefore, $M^* = F_p(\Gamma^*)$ and $X^* \simeq \Gamma^*/M^* \in f(p)$. Thus, $G \simeq X^*/X \in f(p)$. Contradiction.

The theorem is proved.

<u>THEOREM 3.5.</u> Suppose $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ are local formations. If each group in $\mathfrak{U}$ has a $\pi(\mathfrak{F}_2)$-solvable $\mathfrak{F}_2$-coradical, then the local screen f such that

$$f(p) = \begin{cases} \varnothing, & \text{if } p \in \pi'(\mathfrak{F}_1), \\ \mathfrak{N}_p(\mathfrak{F}_1 *_5 \mathfrak{F}_2), & \text{if } p \in \pi(\mathfrak{F}_1), \end{cases}$$

for each prime p is a maximal inner local screen of the formation $l\,form(\mathfrak{F}_1 *_5 \mathfrak{F}_2)$.

<u>Proof.</u> Suppose p is a prime. If $p \in \pi'(\mathfrak{F}_1)$, then $\mathfrak{F}_1 *_5 \mathfrak{F}_2 = \varnothing$, hence $l\,form(\mathfrak{F}_1 *_5 \mathfrak{F}_2) = \mathfrak{E}'$. Consequently, $f(p) = \varnothing$, where f is a maximal inner local screen of $l\,form(\mathfrak{F}_1 *_5 \mathfrak{F}_2)$.

Assume that $p \in \pi(\mathfrak{F}_1)$. Then, obviously, $f(p) \subseteq \mathfrak{N}_p(\mathfrak{F}_1 *_5 \mathfrak{F}_2)$. We will prove that $\mathfrak{N}_p(\mathfrak{F}_1 *_5 \mathfrak{F}_2) \subseteq f(p)$. Choose in the class $\mathfrak{N}_p(\mathfrak{F}_1 *_5 \mathfrak{F}_2) \setminus f(p)$ a group G of least order. Obviously, $O_p(G) = 1$ and $G \in \mathfrak{F}_1 *_5 \mathfrak{F}_2$. Consider the regular wreath product $\Gamma = P \wr G$, where $|P| = p$. Then $\Gamma = N \leftthreetimes G$, where N is an elementary Abelian p-group. Obviously, $N = F_p(\Gamma)$. Suppose F is an $\mathfrak{F}_2$-projector of Γ. Since $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$, it follows from Lemma 3.1 that $F^{f_2(p)} \subseteq F^{f_1(p)}$, where f_1, f_2 are maximal inner local screens of $\mathfrak{F}_1$, $\mathfrak{F}_2$, respectively. Consequently, by Theorem 15.7 of [5] and Lemma 2 of [24], FN is an $\mathfrak{F}_2$ -projector of Γ. Since f_1 is a complete screen, $FN \in f_1(p)$. Moreover, $\Gamma/N \simeq G \in \mathfrak{F}_1 *_5 \mathfrak{F}_2$ and FN covers N. Therefore, it is easy to see that FN is a $\mathfrak{D}\mathfrak{M}$ -subgroup of Γ. Thus, $\Gamma \in \mathfrak{F}_1 *_5 \mathfrak{F}_2 \subseteq l\mathrm{form}(\mathfrak{F}_1 *_5 \mathfrak{F}_2)$. Therefore, $G \simeq \Gamma/F_p(\Gamma) \in f(p)$. Contradiction.

The theorem is proved.

4. Criteria for Localness of Formation Products

THEOREM 4.1. Suppose $\mathfrak{U} = \mathfrak{T}$ and $\mathfrak{F} \subseteq \mathfrak{H}$ are certain formations, where $\mathfrak{F}$ is local. Then $\mathfrak{F} *_1 \mathfrak{H}$ is local if and only if $\mathfrak{F} *_1 \mathfrak{H} = \mathfrak{T}$.

THEOREM 4.2. Suppose each group in $\mathfrak{U}$ has a solvable $\mathfrak{F}$ -coradical, where $\mathfrak{F}$ is a formation with maximal inner local screen ψ. If $\mathfrak{H}$ is a formation such that $\mathfrak{F} \subseteq \mathfrak{H} = \mathfrak{N}_p \mathfrak{H} \subseteq \mathfrak{F} *_2 \psi(p)$ for all p in $\pi(\mathfrak{F})$ then $\mathfrak{F} *_2 \mathfrak{H}$ is local if and only if $\mathfrak{F} *_2 \mathfrak{H} = \mathfrak{U}$.

THEOREM 4.3. Suppose $\mathfrak{U} = \mathfrak{T}$ and $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ are certain local formations, where $\mathfrak{F}_1 \supseteq \mathfrak{N}$. Then $\mathfrak{F}_1 *_5 \mathfrak{F}_2$ is local if and only if $\mathfrak{F}_1 *_5 \mathfrak{F}_2 = \mathfrak{T}$.

The proofs of Theorems 4.1–4.3 are of the same type and follow easily from Theorems 3.1, 3.2, 3.5, respectively, and Lemma 3.1.

THEOREM 4.4. Suppose $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ are formations and f_1, f_2 are maximal inner local screens of $\mathfrak{F}_1$, $\mathfrak{F}_2$ respectively. If each group in $\mathfrak{U}$ has a solvable $f_1(p)$ -coradical for all primes p in $\pi(\mathfrak{F}_1)$, then $\mathfrak{F}_1 *_3 \mathfrak{F}_2$ is local if and only if $\mathfrak{N}_p(\mathfrak{F}_1 *_3 \mathfrak{F}_2) = \mathfrak{F}_1 *_3 \mathfrak{F}_2$ for all primes p such that $f_1(p) \neq \mathfrak{F}_1$.

Proof. If $\mathfrak{F}_1 *_3 \mathfrak{F}_2$ is local, then, by Theorem 3.3, $\mathfrak{N}_p(\mathfrak{F}_1 *_3 \mathfrak{F}_2) = \mathfrak{F}_1 *_3 \mathfrak{F}_2$ for all primes p such that $f_1(p) \neq \mathfrak{F}_1$. Let us prove the converse. Suppose $\mathfrak{N}_p(\mathfrak{F}_1 *_3 \mathfrak{F}_2) = \mathfrak{F}_1 *_3 \mathfrak{F}_2$ for all primes p such that $f_1(p) \neq \mathfrak{F}_1$. Choose in the class $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2) \setminus (\mathfrak{F}_1 *_3 \mathfrak{F}_2)$ a group G of least order. Then G has a unique minimal normal subgroup K. If K is non-Abelian, then $C_G(K) = 1$, hence $C \in f(p)$, where $p \mid |K|$ and f is a maximal inner local screen of $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. By Theorem 3.3, $G \in \mathfrak{F}_1 *_3 \mathfrak{F}_2$, which is impossible. Assume that K is an Abelian p -group, where p is a prime. If $f_1(p) \neq \mathfrak{F}_1$, then $G \in \mathfrak{N}_p(\mathfrak{F}_1 *_3 \mathfrak{F}_2) = \mathfrak{F}_1 *_3 \mathfrak{F}_2$. If $f_1(p) = \mathfrak{F}_1$, then $G \in f(p) = f_2(p) \subseteq \mathfrak{F}_1 *_3 \mathfrak{F}_2$. Contradiction.

The theorem is proved.

THEOREM 4.5. Suppose there exist some set of primes σ and a nonempty formation $\mathfrak{F}$ such that $(\mathfrak{G}_\sigma \cap \mathfrak{U})\mathfrak{F} = \mathfrak{F}$. Suppose $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ are formations and f_1, f_2 are maximal inner local screens of $\mathfrak{F}_1$, $\mathfrak{F}_2$, respectively. If each group in $\mathfrak{U}$ has a solvable $f_1(p)$ -coradical for each $p \in \pi(\mathfrak{F}_1)$ and if either $\mathfrak{F}_1 = \mathfrak{G}_{\sigma'} \cap \mathfrak{U}$ or $\mathfrak{F}_1 = (\mathfrak{G}_{\sigma'} \cap \mathfrak{U})\mathfrak{F}$, then $\mathfrak{F}_1 *_3 \mathfrak{F}_2$ is a local formation.

Proof. If $\mathfrak{F}_1 = \mathfrak{G}_{\sigma'} \cap \mathfrak{U}$, then $\mathfrak{F}_1$ has a maximal inner local screen f_1 such that

$$f_1(p) = \begin{cases} \mathfrak{G}_{\sigma'} \cap \mathfrak{U}, & \text{if } p \in \sigma', \\ \varnothing, & \text{if } p \in \sigma. \end{cases}$$

If $\mathfrak{F}_1 = (\mathfrak{G}_{\sigma'} \cap \mathfrak{U})\mathfrak{F}$, then, by Lemma 3.6, the local screen f_1 such that

$$f_1(p) = \begin{cases} (\mathfrak{G}_{\sigma'} \cap \mathfrak{U})\mathfrak{F}, & \text{if } p \in \sigma', \\ \mathfrak{F}, & \text{if } p \in \sigma, \end{cases}$$

for each prime p is a maximal inner local screen of $\mathfrak{F}_1$. It is obvious that in each of the indicated cases $f_1(p) = \mathfrak{F}_1$ if $p \in \sigma'$, and $f_1(p) = \mathfrak{F}_1 *_2 f_1(p)$ if $p \in \sigma$. Let $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2)$ be the local formation generated by $\mathfrak{F}_1 *_3 \mathfrak{F}_2$, and let f be a maximal inner local screen of this local formation. Let G be a group of least order in the class $l\mathrm{form}(\mathfrak{F}_1 *_3 \mathfrak{F}_2) \setminus (\mathfrak{F}_1 *_3 \mathfrak{F}_2)$. Then G has a unique minimal normal subgroup K, which coincides with $G^{(\mathfrak{F}_1 *_3 \mathfrak{F}_2)}$. Let f_1^* be the local screen such that $f_1^*(p) = \mathfrak{F}_1 *_2 f_1(p)$ for each prime p. If K is f_1^*-eccentric in G, the theorem is obvious. Assume that K is f_1^*-central in G. Then, by Lemma 3.1, $G/C_G(K) \in f_1^*(p) \subseteq f_2(p)$ for some prime p in σ dividing $|K|$. By Lemma 3.4, $G \in \mathfrak{F}_1 *_3 \mathfrak{F}_2$ which is impossible. If $p \in \sigma$, then $f_1(p) = \mathfrak{F}_1$ and, by Theorem 3.3, $G/C_G(K) \in f_2(p)$. Therefore, by Lemma 3.4, $G \in \mathfrak{F}_1 *_3 \mathfrak{F}_2$. Contradiction.

The theorem is proved.

THEOREM 4.6. Suppose $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ are formations and f_1, f_2 are maximal inner local screens of $\mathfrak{F}_1$, $\mathfrak{F}_2$, respectively. Suppose $\mathfrak{U}$ coincides with the set of all $\pi(\mathfrak{F}_2)$-solvable groups in $\mathfrak{G}$. Then $\mathfrak{F}_1 *_4 \mathfrak{F}_2$ is local if and only if $\mathfrak{N}_p(\mathfrak{F}_1 *_4 \mathfrak{F}_2) = \mathfrak{F}_1 *_4 \mathfrak{F}_2$ for all primes p such that $f_1(p) \neq \mathfrak{F}_1$.

The proof of the theorem is analogous to that of Theorem 4.4.

Following [13], we say that a local formation $\mathfrak{F}_1$ is strongly embedded in some local formation $\mathfrak{F}_2$, and we write $\mathfrak{F}_1 \ll \mathfrak{F}_2$, if in any group G having a solvable $\mathfrak{F}_i$-coradical $(i=1,2)$ an $\mathfrak{F}_1$-projector of G is contained in some $\mathfrak{F}_2$-projector of G.

THEOREM 4.7. Suppose $\mathfrak{F}_1$, $\mathfrak{F}_2$ are local formations and each group in $\mathfrak{U}$ has a solvable $\mathfrak{F}_i$-coradical $(i=1,2)$. If $\mathfrak{F}_1 \ll \mathfrak{F}_2$ and $\mathfrak{F}_1 \supseteq \mathfrak{N}$, then $\mathfrak{N}\mathfrak{F}_2$ is the unique maximal (with respect to inclusion) local subformation of $\mathfrak{F}_1 *_4 \mathfrak{F}_2$.

Proof. Obviously, $\mathfrak{N}\mathfrak{F}_2$ is a local subformation of $\mathfrak{F}_1 *_4 \mathfrak{F}_2$. Suppose $\mathfrak{F}$ is an arbitrary local formation contained in $\mathfrak{F}_1 *_4 \mathfrak{F}_2$ with maximal inner local screen f. In view of Lemmas 3.1 and 3.2, to prove the theorem it suffices to show that $f(p) \subseteq \mathfrak{N}_p\mathfrak{F}_2$ for each prime p. Assume this is not so. Let G be a group of least order in the class $f(p) \setminus \mathfrak{N}_p\mathfrak{F}_2$, where p is some prime. Then G has a unique minimal normal subgroup K, which coincides with $G^{\mathfrak{N}_p\mathfrak{F}_2}$. Since $\mathfrak{N}_p\mathfrak{F}_2$ is a saturated formation, K has a complement M in G and $K = C_G(K)$. Since $M^{\mathfrak{F}_1}$ is solvable, it follows from Theorem 15.7 of [5] that M possesses an $\mathfrak{F}_1$-projector F. Obviously, $O_p(G) = 1$. Suppose K is an q-group for some prime $q \neq p$. Then, by Lemma 2 of [24], the subgroup $FC_K(F^{f_1(q)})$ is an $\mathfrak{F}_1$-projector of G. We consider two cases.

1. $C_K(F^{f_1(q)}) = 1$.

Since $M \subset G$, p does not divide $|G:M|$, and $M_G = 1$, it follows from Lemma 2.2 of [13] that G possesses a faithful irreducible G-module N over a field of q elements such that the restriction $N/_M$ of the module M to N has a quotient module N/N_0 on which M acts identically. Let $\Gamma = N \leftthreetimes G$. Since $F \subseteq M$ and $N/_M$ has a quotient module on which M acts identically, we have $[F^{f_1(p)}, N] \subset N$. Suppose F^* is an $\mathfrak{F}_1$-projector of FN. Then, by Lemma 2 of [24] and Theorem 21.10 of [5], $F^* \cap N \neq 1$. Obviously, F^* is an $\mathfrak{F}_1$-projector of Γ. Since $\Gamma \in \mathfrak{F}_1 *_4 \mathfrak{F}_2$, it follows that $F^* \subseteq H$, where H is some $\mathfrak{F}_2$-normalizer of Γ. By Theorem 21.1 of [5], H either covers N or avoids N. If H convers N, then, by Theorem 21.1 of [5], N is $\mathfrak{F}_2$-central Γ, hence $G \simeq \Gamma/N \in f_2(p) \subseteq \mathfrak{N}_p\mathfrak{F}_2$, which is impossible. If H avoids N, then $F^* \cap N = 1$, which contradicts the fact that $F^* \cap N \neq 1$.

2. $C_K(F^{f_1(\varphi)}) \neq 1$.

Obviously, $FC_K(F^{f_1(\varphi)}) \subseteq H^*$, where H^* is some $\mathfrak{F}_2$ -normalizer of G. By Theorem 21.1 of [5], H^* either covers or avoids K. If H^* covers K, then, by Theorem 21.1 of [5], K is $\mathfrak{F}_2$ -central in G, hence $G \in \mathfrak{N}_p \mathfrak{F}_2$. Contradiction. If H^* avoids K, then $C_K(F^{f_1(\varphi)})=1$ and the theorem is true by virtue of part 1.

The theorem is proved.

COROLLARY 4.7.1. Suppose each group in $\mathfrak{M}$ has a solvable $\mathfrak{F}_i$ -coradical, where $\mathfrak{F}_i$ is a local formation $(i=1,2)$, $\mathfrak{F}_1 \prec \mathfrak{F}_2$ and $\mathfrak{F}_1 \supseteq \mathfrak{N}$. Then $\mathfrak{F}_1 *_4 \mathfrak{F}_2$ is local if and only if $\mathfrak{F}_1 *_4 \mathfrak{F}_2 = \mathfrak{N}\mathfrak{F}_2$.

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