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INVARIANT SUBSPACES OF THE ONE-PARAMETRIC GROUP OF SIMILARITIES OF THE MINKOWSKI SPACE

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Lorentzian manifold (M,g) is called self-similar if it admits an essential one-parameter similarity group. It was shown in [1] that in order to find all left-invariant metrics on Lie group $\mathcal{G}$ for which it is a self-similar manifold, we should find Lorentz scalar product in the corresponding Lie algebra $\mathcal{G}$ that admits a one-parameter group of autosimilarities of $\mathcal{G}$.

Suppose that Lie algebra G contains finite number of ideals with certain property (commutative or not commutative). Then these ideals must be invariant under the action of one-parameter similarity group $h(t)$. Therefore, the following problem is of interest: find all invariant subspaces of group $h(t)$. The purpose of this work is to find all two-dimensional invariant subspaces of one-parameter similarity group of the four-dimensional Minkowski space.

Material and methods. We consider a one-parameter group of similarities of the Minkowski space, which has more than one invariant isotropic direction and find its invariant two-dimensional subspaces. Methods of linear algebra and analytical geometry are used.

Results and its discussion. If one-parameter similarity group $h_1(t)$ of the Minkowski space has more than one invariant isotropic direction, then according to [2], it is defined in an appropriate basis (e_1, e_2, e_3, e_4) by the matrix $e^{\mu t}F_1(t)$, $\mu \geq 0$, where

$$F_1(t) = \begin{pmatrix} e^{\nu t} & 0 & 0 & 0 \\ 0 & e^{-\nu t} & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{Q(t)}} \\ 0 & 0 & & \end{pmatrix}, \; t \in \boldsymbol{R},\ \nu > 0,$$

and $Q(t)$ is an orthogonal matrix. In this case, the Gram matrix of the basis has the form

$$\Gamma_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Vectors e_1 and e_2 define invariant directions. The location of the basis vectors relative to the cone of isotropic vectors is shown in figure 1.

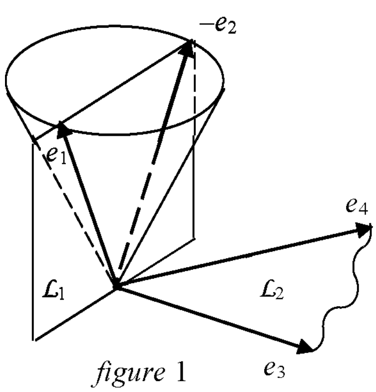

figure 1

We should consider two cases: 1) the matrix $Q(t)$ is not constant; 2) the matrix $Q(t)$ is constant. In the first case, this matrix has the form

$$Q(t) = \begin{pmatrix} \cos at & -\sin at \\ \sin at & \cos at \end{pmatrix}, \; a \neq 0, \qquad (1)$$

and in the second case, $Q(t)$ is the unit matrix. Linear span of vectors x, y we denote $\langle x, y \rangle$.

***Theorem.* 1.** *Subspaces* $\mathcal{L}_1 = \langle e_1, e_2 \rangle$, $\mathcal{L}_2 = \langle e_3, e_4 \rangle$, *and only they are two-dimensional subspaces invariant under the action of the one-parameter group* $h_1(t)$, $t \in \boldsymbol{R}$, *if matrix* $Q(t)$ *has form* (*1*).

2. *All two-dimensional subspaces contained in the three-dimensional subspaces* $\mathcal{H}_1 = \langle e_2, e_3, e_4 \rangle$ *and* $\mathcal{H}_2 = \langle e_1, e_3, e_4 \rangle$, *as well as the subspace* L_1, *and only they are invariant under the action of the one-parameter group* $h_1(t)$, $t \in \boldsymbol{R}$, *if* $\nu > 0$ *and* $Q(t) = E$.

Proof. Consider one-parameter isometry group $f_1(t)$, which is given by the matrix $F_1(t)$. It has the same invariant subspaces as $h_1(t)$. Consider an arbitrary vector $X(x_1, x_2, x_3, x_4)$ and denote $X'(t) = f_1(t)X$, $X''(t) = f_1(t)X'(t)$. Vector X belongs to an invariant two-dimensional subspace if and only if the vectors $X, X'(t), X''(t)$ are linearly dependent for any $t \in \boldsymbol{R}$. The subspace invariant for all $t \in \boldsymbol{R}$, is invariant for each fixed value $t = t_0$.

Let matrix $Q(t)$ have form (1). It is required to prove that there are no other invariant subspaces except $\langle e_1, e_2\rangle$ and $\langle e_3, e_4\rangle$. We choose $t_0 = \dfrac{\pi}{2a}$ and denote $k = e^{\nu t_0} > 1$. Then

$$X'(t_0)(kx_1, k^{-1}x_2, -x_4, x_3),\ \ X''(t_0)(k^2x_1, k^{-2}x_2, -x_3, -x_4).$$

Let's compose a matrix from the coordinates of vectors $X, X'(t_0), X''(t_0)$ and then add the first row to the third row. We get matrix

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ kx_1 & k^{-1}x_2 & -x_4 & x_3 \\ (k^2+1)x_1 & (k^{-2}+1)x_2 & 0 & 0 \end{pmatrix}.$$

We calculate the minor of order 3, located in 1, 3 and 4 columns equate it to zero. We get

$$(k^2+1)x_1(x_3^2+x_4^2) = 0.$$

Hence, $x_1 = 0$ or $x_3^2 + x_4^2 = 0$, and this holds regardless of the value of ν. Analogously, calculating the minor located in 2, 3 and 4 columns, we get $x_2 = 0$ or $x_3^2 + x_4^2 = 0$. As a result, we have only two invariant two-dimensional subspaces $\langle e_1, e_2\rangle$ and $\langle e_3, e_4\rangle$.

Now let the matrix $Q(t)$ be unit one and $\nu > 0$. Then

$$X'(t)(e^{\nu t}x_1, e^{-\nu t}x_2, x_3, x_4),\ \ X''(t)(e^{2\nu t}x_1, e^{-2\nu t}x_2, x_3, x_4).$$

Let's compose a matrix from the coordinates of vectors X, X', X'', and then subtract the first row from the second and third ones. We get matrix

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ (e^{\nu t}-1)x_1 & (e^{-\nu t}-1)x_2 & 0 & 0 \\ (e^{2\nu t}-1)x_1 & (e^{-2\nu t}-1)x_2 & 0 & 0 \end{pmatrix}$$

Equating to zero the minor located in the first three columns, we obtain equation

$$(e^{-\nu t}-1)x_1x_2x_3 = 0 \quad \Leftrightarrow \quad x_1x_2x_3 = 0.$$

Analogously, equating to zero the minor located in columns 1, 2, and 4, we get $x_1x_2x_4 = 0$. Therefore, either $x_3 = x_4 = 0$, or $x_1x_2 = 0$. The first equation defines subspace L_2. Equation $x_1 = 0$ defines $\mathcal{H}_1$, and equation $x_2 = 0$ defines $\mathcal{H}_2$. This means that any two-dimensional subspace contained in $\mathcal{H}_1$ or $\mathcal{H}_2$, is invariant under the action $f_1(t)$, $t \in \boldsymbol{R}$.

Corollary. Subspace $\mathcal{L}_1 = \langle e_1, e_2\rangle$ and only it is a two-dimensional subspace invariant under the action of the one-parameter group $f_1(t)$, $t \in \boldsymbol{R}$, on which the Lorentzian scalar product is induced.

Conclusion. The proved theorem essentially complements the results obtained in (5). It is used to find one-parameter groups of automorphisms of four-dimensional Lie algebras, which are similarities with respect to the Lorentz scalar product introduced in the Lie algebra.

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ИЕРАРХИЧЕСКАЯ КЛАСТЕРИЗАЦИЯ

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Очень часто в прикладных задачах встречаются случаи, когда нужно изучить структуру данных, найти какие-либо взаимосвязи и иерархию. Для этих целей можно использовать различные методы: методы понижения размерности, которые основаны на матричных разложениях и др. Такие методы позволяют сохранить старые данные на основе новых.