The main result is the following

Theorem. Let M be a semiformation and $A \in l^{\sigma}$formM. If $O_{\sigma_i}(A) = 1$, and $\sigma_i \in \sigma$, then $A \in l^{\sigma}$formM$_1$, where M$_1$ = $\{G / O_{\sigma_i}(G) \mid G \in$ M$\}$.

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ON MODULARITY OF THE LATTICE OF σ-LOCAL FITTING CLASSES

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All groups considered are finite. In definitions and notations we follow N.N. Vorob’ev “Algebra of Classes of Finite Groups" and K. Doerk "Finite Soluble Groups” [1, 2].

The problem of describing the properties of the lattice of classes of groups and subgroup systems is the actual problem of the group theory. In this field of research, we know the result of A.N. Skiba [3] on the modularity of the lattice of formations of groups. However, the question of the modularity of the lattice of Fitting classes and lattice of Fitting sets of group remains open to date (see [4, Problem 14.47]).

The purpose of this paper is to prove the modularity of the lattice of σ-local Fitting classes.

Material and methods. The lattice is a partially ordered set, in which each two-element subset has both an exact top and an exact bottom edge. A lattice L is called modular if for any $x, y, z \in L$ such as $x \leq y$ the following equality holds $x \vee (y \wedge z) = y \wedge (x \vee z)$, called a modular law.

The non-empty class of groups $\mathfrak{F}$ is called the Fitting class, if and only if the following conditions are fulfilled:

1) if $G \in \mathfrak{F}$ и $N \trianglelefteq G$, then $N \in \mathfrak{F}$;
2) if $M, N \in \mathfrak{F}$, $M \trianglelefteq G, N \trianglelefteq G$ и $G = MN$, then $G \in \mathfrak{F}$.

The symbol $G^{\mathfrak{F}}$ denotes the intersection of all normal subgroups N such that $G/N \in \mathfrak{F}$.

We write $\mathfrak{G}_{\sigma_i}$ to denote the class of all σ_i-groups and $\mathfrak{G}_{\sigma'_i}$ to denote the class of all σ'_i-groups.

Following Shemetkov [5], σ is some partition of $\mathbb{P}$, that is, $\sigma = \{\sigma_i | i \in I\}$, where $\mathbb{P} = \cup_{i \in I} \sigma_i$, $\sigma_i \cap \sigma_j = \emptyset$ for all $i \neq j$.

We call the function of the form

$$f: \sigma \longrightarrow \{Fitting\ classes\}$$

Hartley σ-function (or simply H_σ-function), and we put

$$LR_\sigma(f) = \left(G \middle| G = 1 \text{ или } G \neq 1 \text{ и } G^{\mathfrak{G}_{\sigma_i}\mathfrak{G}_{\sigma'_i}} \in f(\sigma_i) \text{ для всех } \sigma_i \in \sigma(G)\right).$$

In this paper we use methods of abstract group theory, in particular methods of classes theory and lattice theory.

Findings and their discussion.

We can show, that

$$\big(f_1(\sigma_i) \vee f_2(\sigma_i)\big) \cap f_3(\sigma_i) = f_1(\sigma_i) \vee \big(f_2(\sigma_i) \cap f_3(\sigma_i)\big),$$

$$\forall \sigma_i : f_1(\sigma_i) \subseteq f_3(\sigma_i) \Leftrightarrow \mathfrak{F}_1 \subseteq \mathfrak{F}_3,\ i \in I.$$

Let $\mathfrak{X}$ be some set. We write $Fit(\mathfrak{X})$ to denote the intersection of all Fitting classes, that includes $\mathfrak{X}$.

Let $\mathfrak{F}$ and $\mathfrak{H}$ be σ-local Fitting classes. Then:

$$\mathfrak{F} \vee_\sigma \mathfrak{H} = Fit(\mathfrak{F} \cup \mathfrak{H}).$$

Let $\mathfrak{F}_i = LR_\sigma(f_i)$, where f_i – H_σ-functions of the respective classes $\mathfrak{F}_i$. Then:

1) $\vee_{\sigma\, i \in I} \mathfrak{F}_i = LR_\sigma(\vee_{i \in I} f_i)$,

2) $\cap_{i \in I} \mathfrak{F}_i = LR_\sigma(\cap_{i \in I} f_i)$.

We proved

Theorem. Let $\mathfrak{F}$ and $\mathfrak{H}$ be σ-local Fitting classes. If $\mathfrak{X}$ is σ-local Fitting class and $\mathfrak{F} \subseteq \mathfrak{X}$, then $(\mathfrak{F} \vee \mathfrak{H}) \cap \mathfrak{X} = \mathfrak{F} \vee (\mathfrak{H} \cap \mathfrak{X})$.

Conclusion. The work shows the modularity of the lattice of σ-local Fitting classes.

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