In the theory of classes of groups the know result of Lockett [2] that the product two of Fischer classes is Fischer class.

In [3] it is defined the product Fitting set and Fitting class.

Definition [3]. For a Fitting set $\mathcal{F}$ of G and a Fitting class $\mathfrak{F}$, we call the set $\{H \leq G | H/H_{\mathcal{F}} \in \mathfrak{F}\}$ of subgroups of G *the product of* $\mathcal{F}$ *and* $\mathfrak{F}$, and denote it by $\mathcal{F} \circ \mathfrak{F}$.

As stated in [3] $\mathcal{F} \circ \mathfrak{F}$ is a Fitting class.

The main purpose of this paper study of the product of Fischer set and Fischer class. It is proved.

Theorem. Let $\mathcal{F}$ is a Fischer set of G and $\mathfrak{F}$ is a Fischer class, then $\mathcal{F} \circ \mathfrak{F}$ is a Fischer set of G.

Reference list:

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3. Vorob'ev N.T. On F-injectors of Fitting set of a finite group / Vorob'ev N.T., Nanying Yang, W.Guo. – Com. in Algebra, 2018, Vol. 46, № 1. – P. 217–229.

ON SOLVABILITY OF SOME CLASSES OF ALGEBRAIC EQUATIONS IN RADICALS

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The relevance of this work is to obtain the most convenient method for solving algebraic equations, and to answer the question whether this equation is solvable in radicals.

The purpose of the article is formulate and justify the necessary and sufficient conditions for the representability of algebraic polynomials of the fourth and eighth degree in the form of a superposition of quadratic polynomials, as a consequence of the solvability conditions in the eighth degree equation by radicals.

Material and methods. Material of this article the algebraic equations. In the study, methods of algebra, mathematical analysis and the computer mathematics system Maple 2017 were used in the research.

Findings and their discussion. Suppose the algebraic equations of the form:

$$f(z) = 0,$$

where $f(z)$ - fourth degree polynomial, which is a superposition of a quadratic polynomial. That is, has the following form:

$$f(z) = (z^2 + b_1 z + c_1)^2 + b_2(z^2 + b_1 z + c) + c_2 .$$

Opening the brackets, we get:

$$f(z) = z^4 + 2b_1 z^3 + \left(b_1^2 + b_2 + 2c_1\right) z^2 + \left(b_2 b_1 + 2c_1 b_1\right) z + b_2 c_1 + c_1^2 + c_2 . \qquad (1)$$

Find out under what conditions a polynomial of the fourth degree of the form:

$$f(z) = z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 , \qquad (2)$$

is a superposition of a quadratic polynomial. To do this, we analyze the system of equations arising after equating the coefficients of the polynomial (1) with the coefficients of the polynomial (2):

$$b_1 = \frac{a_1}{2};$$

$$b_2 = a_2 - \frac{a_1^2}{4} - 2c_1;$$

$$a_3 = -\frac{1}{8} a_1^3 + \frac{1}{2} a_1 a_2 .$$

It follows that the coefficient a_3 is associated with the coefficients a_1 and a_2. Similarly, we express the coefficient a_4:

$$a_4 = a_2 c_1 - \frac{a_1^2}{4} c_1 - c_1^2 + c_2 .$$

We get the following identity:

$$\left(z^2 + \frac{a_1}{2} z + c_1\right)^2 + \left(a_2 - \frac{a_1^2}{4} - 2c_1\right)\left(z^2 + \frac{a_1}{2} z + c_1\right) + \frac{a_1^2 c_1}{4} + c_1^2 - a_2 c_1 \equiv$$

$$\equiv z^4 + a_1 z^3 + a_2 z^2 + \frac{a_1}{2}\left(a_2 - \frac{a_1^2}{4}\right) z + a_4 . \qquad (3)$$

Identity (3) follows a necessary and sufficient condition for the representation of a fourth-degree polynomial as a superposition of a quadratic polynomial, that is, the following theorem holds.

Theorem 1. A necessary and sufficient condition for the representation of a polynomial (2) as a superposition of a quadratic polynomial is the following condition:

$$a_3 = \frac{a_1}{2}\left(a_2 - \frac{a_1^2}{4}\right),$$

expressing the relationship between the coefficients of the polynomial (2).

Similar reasoning can be applied to polynomials of the eighth degree, which is a superposition of a square polynomial of the fourth degree. Polynomial has the form:

$$\begin{aligned} f(z) = {} & z^8 + 4b_1z^7 + (6b_1^2 + 2b_2 + 4c_1)z^6 + (2b_1b_2 + 4c_1b_1 + 4(b_1^2 + b_2 + \\ & + 2c_1)b_1)z^5 + (b_3 + 2b_2c_1 + 2c_1^2 + 2c_2 + 4(b_1b_2 + 2c_1b_1)b_1 + (b_1^2 + b_2 + \\ & + 2c_1)^2)z^4 + (2b_3b_1 + 4(b_2c_1 + c_1^2 + c_2)b_1 + 2(b_1b_2 + 2c_1b_1)(b_1^2 + b_2 + \\ & + 2c_1))z^3 + (b_3(b_1^2 + b_2 + 2c_1) + 2(b_2c_1 + c_1^2 + c_2)(b_1^2 + b_2 + 2c_1) + \\ & + (b_1b_2 + 2c_1b_1)^2)z^2 + (b_3(b_1b_2 + 2c_1b_1) + 2(b_2c_1 + c_1^2 + c_2)(b_1b_2 + \\ & + 2c_1b_1))z + b_3(b_2c_1 + c_1^2 + c_2) + (b_2c_1 + c_1^2 + c_2)^2 + c_3. \end{aligned} \tag{4}$$

Find out under what conditions a polynomial of the eighth degree, which is a superposition of three quadratic polynomials. Such a superposition is as follows:

$$f(z) = z^8 + a_1z^7 + a_2z^6 + a_3z^5 + a_4z^4 + a_5z^3 + a_6z^2 + a_7z + a_8. \tag{5}$$

To do this, we will analyze the system resulting from equating the coefficients of the polynomial (3) and the coefficients of the polynomial (4):

$$a_3 = \frac{7}{32}a_1^3 + \frac{3}{4}a_1a_2; \tag{6}$$

$$a_5 = \frac{1}{256}a_1(7a_1^4 - 20a_1^2a_2 + 128a_4); \tag{7}$$

$$a_6 = -\frac{7}{4096}a_1^6 + \frac{1}{256}a_1^4a_2 + \frac{3}{64}a_1^2a_2^2 - \frac{1}{8}a_1^2a_4 + \frac{1}{2}a_2a_4 - \frac{1}{8}a_2^3; \tag{8}$$

$$a_7 = -\frac{1}{2048}a_1(3a_1^2 - 8a_2)(a_1^4 - 8a_2^2 + 32a_4). \tag{9}$$

The following theorem holds.

Theorem 2. A necessary and sufficient condition for representing a polynomial (5) as a superposition of a quadratic square polynomial is conditions (6) - (9)

Example.

Given the fourth degree equation:

$$z^4 + 5z^3 + 4z^2 - \frac{45}{8}z + 1 = 0.$$

It is necessary to find the solutions of this equation. Using the method given earlier, we represent the equation in the form (1).

Find the coefficients b_1, b_2, c_1, c_2. We will substitute the obtained coefficients into system (2) and solve it. We obtain the following roots of a fourth-degree equation:

$$z_1 = -\frac{5}{4} + \frac{\sqrt{43 + 2\sqrt{17}}}{4},\quad z_2 = -\frac{5}{4} - \frac{\sqrt{43 + 2\sqrt{17}}}{4},\quad z_3 = -\frac{5}{4} + \frac{\sqrt{43 - 2\sqrt{17}}}{4},$$

$$z_4 = -\frac{5}{4} - \frac{\sqrt{43 - 2\sqrt{17}}}{4}.$$

Conclusion. As a result of the study, necessary and sufficient conditions for fourth and eighth degree polynomials were found, in the form of a superposition of two and three second degree polynomials.