COMPUTER ALGEBRA SYSTEM GAP IN THE THEORY OF SCHUNCK CLASSES

I. Staselka
VSU named after P.M. Masherov, Vitebsk, Belarus

The aim of this talk to find an **algorithm** for the computation of F-projectors for arbitrary Schunck classes F given by an algorithm testing membership in F (or even only in the basis or boundary of F) using GAP package CRISP.

Computer algebra system GAP – Groups, Algorithms and Programming – initially has been developed as a tool for Combinatorial Group Theory. Further GAP began to be used in other areas of algebra (the Theory of Finite Soluble Groups, the Theory of Formations and the Theory of Schunck Classes). In particular in 2000 GAP package CRISP was developed by Burkhard Höfling. CRISP stands for Computing with Radicals, Injectors, Schunck classes and Projectors of finite soluble groups. All groups considered are finite and soluble.

Recall that F is a Schunck class if it consists of all groups *G* such that $G/Core_G(M) \in$ F for every maximal subgroup *M* of *G* and note that a Schunck class is closed with respect to factor groups. A group *G* having a maximal subgroup *M* with trivial core will be called primitive.

Let F be a class of groups. A subgroup *H* of the group *G* is an F-maximal subgroup of *G* if it belongs to the class F but is not properly contained in another subgroup of *G* belonging to F. F-projectors are examples of such maximal subgroups. A subgroup *H* of a group *G* is an F-projector of *G* if *HN*/*N* is F-maximal in *G*/*N* for every normal subgroup *N* of *G*. Projectors play an important role in the theory of finite soluble groups and have been studied intensively (see [1]). In particular, projectors exist in every group if and only if F is a Schunck class. In the special case when the Schunck class F is also a local formation, an algorithm for computing F-projectors of a group *G* has been described in Eick and Wright [2]. However, the Eick-Wright algorithm essentially uses the description of local formations by means of functions called inner local satellites (see [3,4]).

Let F be a Schunck class. It is clear that the class F is completely determined by the primitive groups in F. This class of primitive groups belonging to F is called the basis of F. A Schunck class F can also be represented by its boundary, which consists of all groups *G* such that $G \notin$ F but every proper homomorphic image of *G* belongs to F.

Reference list:

1. Doerk, K. Finite Soluble Groups / K. Doerk, T. Hawkes. – Berlin-New York : Walter de Gruyter & Co., 1992. – 891 p. – (De Gruyter Expo. Math., vol. 4).

2. Eick, B. Computing subgroups by exhibition in finite solvable groups / B. Eick, C.R.B. Wright // J. Symbolic Computation. – 2002. – Vol. 33: – P. 129–143.

3. Shemetkov, L.A. Formations of finite groups / L.A. Shemetkov. – Moscow: Nauka, 1978. – 272 p.
4. Shemetkov, L.A. Formation of algebraic systems / L.A. Shemetkov, A.N. Skiba. – Moscow: Nauka, 1989. – 256 p.

ABOUT THE PRODUCT FISHER SET AND FISHER CLASS

H. Vaitkevich
VSU named after P.M. Masherov, Vitebsk, Belarus

Throughout this paper, all groups are finite. In the definitions and notation, we follow [1].

Remind that class $\mathfrak{F}$ is a *Fitting class* [1] if and only if the following two conditions are satisfied:

(i) If $G \in \mathfrak{F}$ and $N \trianglelefteq G$, then $N \in \mathfrak{F}$;

(ii) If $M, N \trianglelefteq G = MN$ with M and N in $\mathfrak{F}$, then $G \in \mathfrak{F}$.

A subgroup U of a group G is said to be *subnormal in* G if there exists a chain of subgroups $U_0, U_1, \dots, U_r$ of G such that

$U = U_0 \trianglelefteq U_1 \trianglelefteq \cdots \trianglelefteq U_{r-1} \trianglelefteq U_r = G.$

This is called a *subnormal chain from* U *to* G. If U is subnormal in G, we shall write $U \trianglelefteq\trianglelefteq G$.

Theorem [1]. Let $\{U_i : i \in I\}$ be a *set of subnormal subgroups of a finite group G*. Then their join $\mathcal{J} =< U_i : i \in I >$ is also subnormal in G.

For a class $\mathfrak{F}$ of groups we define:

$N_0 \mathfrak{F} = (G: \exists K_i \trianglelefteq\trianglelefteq G \ (i = 1, \dots, r) \ with \ K_i \in \mathfrak{F} \ and \ G =< K_1, \dots, K_r >).$

A class $\mathfrak{F}$ of arbitrary finite groups is called a *Fischer class* [1] if

(i) $\mathfrak{F} = N_0 \mathfrak{F} \neq \emptyset$, and

(ii) If $K \trianglelefteq G \in \mathfrak{F}$ and H/K is a nilpotent subgroup of G/K, then $H \in \mathfrak{F}$.

A non-empty set $\mathcal{F}$ of subgroups of a group G is called a *Fitting set of* G [1] if the following three conditions are satisfied:

FS1: If $T \trianglelefteq\trianglelefteq S \in \mathcal{F}$, then $T \in \mathcal{F}$;

FS2: If $S, T \in \mathcal{F}$ and $S, T \trianglelefteq ST$, then $ST \in \mathcal{F}$;

FS3: If $S \in \mathcal{F}$ and $x \in G$, then $S^x \in \mathcal{F}$.

Lemma [1]. *Let* $\mathfrak{F}$ *be an* N_0*-closed class and* G *a finite group. Then the set* $\mathcal{G} = \{N \trianglelefteq\trianglelefteq G : N \in \mathfrak{F}\}$, *partially ordered by inclusion, has a unique maximal element, denoted by* $G_{\mathfrak{F}}$ *and called the* $\mathfrak{F}$*-radical of G. It is a characteristic subgroup of* G, *and if* $\mathfrak{F}$ *is a Fitting class and* $K \trianglelefteq\trianglelefteq G$, *then* $K_{\mathfrak{F}} = K \cap G_{\mathfrak{F}}$.

A *Fischer set of* G [1] is a Fitting set $\mathcal{F}$ of G which has the following property:

FS4: If $K \trianglelefteq L \in \mathcal{F}$ and if H/K is a nilpotent subgroup of L/K, then $H \in \mathcal{F}$.