$L_\pi(\mathfrak{F}) = (G \in \mathfrak{S}$: the $\mathfrak{F}$-injectors of G have π'-index in G).

Thus $L_\pi(\mathfrak{F})$ consists of all finite soluble groups whose $\mathfrak{F}$-injectors contain a Hall π-subgroup: in particular, we have $L_\varnothing(\mathfrak{F}) = \mathfrak{S}$ and $L_P(\mathfrak{F}) = \mathfrak{F}$.

Let $\mathfrak{F}$ and $\mathfrak{H}$ be a Fitting class. Then $\mathfrak{F}\mathfrak{H} = (G$: $G/G_{\mathfrak{F}} \in \mathfrak{H})$. Let $\mathfrak{E}_{\pi'}$ be a class of all π'-groups. In [2], the theorem is proved that if $\mathfrak{F}\mathfrak{E}_{\pi'} = \mathfrak{F}$, then in any π-soluble group there are $\mathfrak{F}$-injectors and any two of them are conjugate.

Doerk and Hawkes in the class of all finite soluble groups obtained a criterion for the permutability of Fitting classes. In connection with it the actual task of extending this result to the case of partially solvable groups. The solution to this question is the main goal of this paper.

Let $\mathfrak{S}^\pi$ be a class of all π-soluble groups.

THEOREM. Let $\mathfrak{F}$ be a Fitting class, let $\pi \subseteq P$, and let $G \in \mathfrak{S}^\pi$. Let V and $G_{\pi'}$ denote respectively an $\mathfrak{F}$ -injector and a Hall π'-subgroup of G, and put $W=\langle V, G_{\pi'}\rangle$. Then W is an $L_\pi(\mathfrak{F})$ -injector of G if and only if $VG_{\pi'} = G_{\pi'}V$.

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THE INVARIANCE OF PROPERTY OF SEPARATION FROM ZERO CORNER MINORS IN THE SIMILARITY TRANSFORMATION USING THE LOWER TRIANGULAR MATRIX

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Exploring characteristics of a numerical matrix in its various matrix transformations (e.g., in the similarity transformation, congruence, etc.) is one of the main problems in matrix theory.

The aim of this paper is to learn preservation of the principal leading corner minors’ of a square n-dimensional matrix positivity and, moreover, of their separation from zero using the similarity transformation on the set of lower triangular matrices with positive diagonal elements.

Material and methods. We obtained the main results using the methods of linear algebra and matrix theory.

Findings and their discussion. Let R^n be a n-dimensional Euclidean vector space supplied with the norm $\|x\| = \sqrt{x^T x}$ (here the symbol T means the transpose of a matrix or a vector); $e_1, e_2, \ldots, e_n$ be vectors (columns) of the canonical orthonormal basis for the space R^n; M_{mn} be the space of real $m \times n$-dimensional matrices supplied with the spectral (operator) norm

$\|H\| = \max\limits_{\|x\|=1} \|Hx\|$, i.e. the norm induced by the Euclidean norm in the spaces $\mathbb{R}^n$ and $\mathbb{R}^m$ [1, p. 357]; $\mathrm{M}_n := \mathrm{M}_{nn}$. Denote by $E = [e_1, \dots, e_n] \in \mathrm{M}_n$ the identity matrix. For any number $l \in \mathbb{N}$ denote the set of lower triangular $l \times l$-matrices with positive diagonal elements by $\mathrm{R}_l \subset \mathrm{M}_l$.

Definition 1. For any fixed number $k \in \{1, \dots, n\}$ and any matrix $H = \{h_{ij}\}_{i,j=1}^{n} \in \mathrm{M}_n$ by $(H)_k \in \mathrm{M}_k$ denote it's *principal leading* k*-dimensional submatrix* [1, p. 30], i.e.

$$(H)_1 = (h_{11}), \quad (H)_2 = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}, \quad \dots, \quad (H)_n = H.$$

Determinants of leading principal submatrices of a matrix $H \in \mathrm{M}_n$ are called the *principal leading corner minors* [1, p. 30].

For any number $l \in \mathbb{N}$ by $\mathrm{H}_l \subset \mathrm{M}_l$ denote the set of l-dimensional matrices with positive principal leading minors, i.e.

$$\mathrm{H}_l := \{H \in \mathrm{M}_l : \det(H)_k > 0, \ k = \overline{1, l}\}.$$

Definition 2. For any number $j = \overline{1, n}$ by $S_j \in \mathrm{M}_n$ denote the matrix received from a matrix R by changing it's first j strings by the appropriate strings of a matrix H, i.e.

$$S_j := R + \sum_{i=1}^{j} e_i e_i^T (H - R), \quad j = \{1, \dots, n\}.$$

In the sequel, with the use of monograph's terminology [2, p. 283], consider matrices $S_j \in \mathrm{M}_n$, $j = \overline{1, n}$, to be the *intermediate steps on the way from* R *to* H.

Definition 3. An ordered pair (R, H) of matrices on the set M_n is called *law-abiding* [2, p. 283], if the ratio $\det R > 0$ is correct and for each number $j \in \{1, \dots, n\}$ of matrices S_j, which are the intermediate steps on the way from R to H, such ratios as $\det S_j > 0$ are correct.

Definition 4. *Square* n*-dimensional matrices* M and N are called *similar* [1, p. 61], if such matrix as $S \, (\det S \neq 0)$ exists so the following equality is correct

$$M = SNS^{-1}$$

and the transformation of matrix N itself with the use of the matrix S is called the similarity transformation.

Theorem 1. *Let* $R \in \mathrm{R}_n$, $H \in \mathrm{H}_n$. *If a pair* (R, H) *is law-abiding, then the positivity of the principal leading minors of the matrix* H *is preserved by the similarity transformation with the use of the matrix* R, *so the inclusion* $RHR^{-1} \in \mathrm{H}_n$ *is performed.*

The invariance of the positivity property of the principal leading corner minors of the matrix $H \in \mathrm{H}_n$ in the similarity transformation using the matrix $R \in \mathrm{R}_n$ is established by theorem 1, if these matrices are law-abiding. However, a stronger statement is also true. It is about the preservation of the principal corner minors' of the matrix H separation from zero in the similarity transformation using with not only the law-abiding condition but also the ρ-law-abiding of the same pair (R,H).

For any real numbers $r \geq 1$ and $\rho \in (0,1)$ let $\mathrm{R}_n(\rho,r) \subset \mathrm{R}_n$ be a set of lower triangular $n \times n$-matrices R with positive diagonal elements for which $\|R-E\| \leq r$ and $\det R \geq \rho$ are correct, i.e.

$$\mathrm{R}_n(\rho,r) := \{R \in \mathrm{R}_n : \|R-E\| \leq r, \det R \geq \rho\},$$

and a set of $n \times n$-matrices $\mathrm{H}_n(\rho,r) \subset \mathrm{M}_n$, for which $\|H-E\| \leq r$ and all the principal leading minors are not less than ρ, i.e.

$$\mathrm{H}_n(\rho,r) := \{H \in \mathrm{M}_n : \|H-E\| \leq r,\ \det(H)_k \geq \rho,\ k=\overline{1,n}\}.$$

<u>Definition 5.</u> Let $\rho \in (0,1)$ be a random fixed number. According to the definition from [2, p. 283] an *ordered pair* (R,H) on a set M_n^2 is *called* ρ*-law-abiding*, if $\det R \geq \rho$ and for any $j \in \{1,\dots,n\}$ belonged to S_j, which are intermediate steps on the way from R to H, such inequalities as $\det S_j \geq \rho$ are correct.

<u>Theorem 2</u>. *Suppose $r \geq 1$ and $\rho \in (0,1)$. If a pair (R,H) for which $R \in \mathrm{R}_n(\rho,r)$ and $H \in \mathrm{H}_n(\rho,r)$ is ρ-law-abiding, then there exist such numbers as $\rho_1 = \rho_1(\rho,r) \in (0,1)$ and $r_1 = r_1(\rho,r) \geq 1$, for which $RHR^{-1} \in \mathrm{H}_n(\rho_1,r_1)$.*

Consclusion. The obtained results can be further used in the theory of controllability of asymptotic invariants of linear systems of ordinary differential equations in the study of global Lyapunov reducibility [2, p. 258 - 259] and even the global attainability [2, p. 253] of such systems.

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ASSESSMENT OF THE MATCHING OF THE RECOMMENDED GAS VELOCITIES IN CYCLONES TO THE ECONOMICALLY OPTIMAL ONES

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Gas cleaning in the chemical industry is a complex and urgent problem with a number of aspects. The economic aspects are the development of the methods and systems that can significantly improve the efficiency of extraction of the valuable components from raw materials, maximize the utilization of the