

Otherwise, there are problems with their reading by the mobile device. This difficulty is easily overcome after several uses of the application.

In our opinion, «the Plickers» program can be used to organize various types of knowledge testing: preliminary (diagnostic) testing, used to study the level of students' readiness for perception of new material; current testing designed to check the learning of the previous material; thematic testing, the purpose of which is to compile and systematize the educational material of the whole topic; final testing, aimed at checking the specific learning outcomes.

Conclusion. Practical use of the mobile application «Plickers» allows you to reduce the time required to survey students, to visualize the results of testing and analyze the level of learning the studied material.

Reference list:

1. Founder and chief executive officer of Plickers. [Electronic resource]. – Access mode: <https://2018.edcrunch.ru/speakers/nolan-emi/> – Access date: 09/06/2018.
2. Plickers is a powerfully simple tool. [Electronic resource]. – Access mode: <https://www.plickers.com> – Access date: 12.09.2018.

ANALYTICAL SOLUTION OF CUBIC MATRIX EQUATIONS WITH COMMUTATIVE MATRIX FACTORS OF SIZE [2X2]

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Nonlinear matrix equations are found in numerous applications. Iterative methods, for example, modification of the method of Newton-Kantorovich and algorithm Bernoulli are often used for their solutions [1–2]. Also, the exact analytical solution for some types of matrix nonlinear equations, in particular, cubic ones, is interesting.

The aim of the research is to present an algorithm for the exact analytical finding of a cubic matrix equation with commutative matrices-coefficients of size [2×2].

Material and methods. The material of the research is cubic matrix equations with commutative coefficients and methods of their solution. Research methods-methods of mathematical and functional analysis.

Findings and their discussion. It is necessary to remind that in the scalar case of the solution of the cubic equation with complex coefficients of the form (1) can be found using the formula Cardano (2) [3, p. 235]:

$$x^3 + px + q = 0 \quad (1)$$

$$x = \alpha + \beta = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}, \quad (2)$$

$$\alpha\beta = -\frac{P}{3}.$$

We will consider the cubic matrix equation (3) in which all matrixes commutative, contain real elements and have the size $[2 \times 2]$:

$$X^3 + PX + Q = 0. \quad (3)$$

Designate

$$D = \frac{Q^2}{4} + \frac{P^3}{27}.$$

This matrix is reduced to the Jordan normal form [4, p. 148]

$$J = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where λ_1 and λ_2 are the eigenvalues of the matrix D ($|\lambda_1| < |\lambda_2|$).

Extract the square roots of the elements of the matrix J :

$$J_1 = \begin{pmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{pmatrix}.$$

Next, you need to write the form of the transition matrix from $D^{1/2}$ to its equivalent matrix J_1 . Recall that the transition matrix T consists of the eigenvectors of the matrix D .

$$D^{1/2} = D_{1/2} = TJ_1T^{-1}.$$

We will designate

$$Y = -\frac{Q}{2} + D_{1/2}$$

matrix expression, similar to the first radicand in a formula (2). It must also be reduced to Jordan normal form J_2 .

Extract the cubic roots of the elements of the matrix J_2 :

$$J_3 = \begin{pmatrix} \sqrt[3]{J_2[1,1]} & 0 \\ 0 & \sqrt[3]{J_2[2,2]} \end{pmatrix}.$$

In the general case, for each of the numbers $J_2[1,1]$ and $J_2[2,2]$, there are 3 different complex values of the cubic root. Therefore, writing out all possible combinations, we get 9 different matrices $J_{3,i}$ ($i=1, \dots, 9$).

Substituting the obtained values $J_{3,i}$ into expression

$$Y_i^{1/3} = \Theta J_{3,i} \Theta^{-1} = \alpha_i \quad (i=1, \dots, 9),$$

find the 9 values of the cubic root of the matrix Y .

Further, for each matrix α_i , we find, by analogy with the scalar case, the matrix β_i :

$$\beta_i = -\frac{P}{3} \alpha_i^{-1} \quad (i=1, \dots, 9).$$

Thus, the solutions of the original matrix equation (3) are of the form

$$X_i = \alpha_i + \beta_i \quad (i=1, \dots, 9).$$

Conclusion. In the course of the work it is shown that all solutions of a random matrix equation with commutative coefficients of size $[2 \times 2]$ can be analytically obtained. Also, the algorithm for finding these solutions is presented.

Reference list:

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FITTING PERMUTATION CLASSES OF π - SOLUBLE GROUPS

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The main goal of the paper is a description of Fitting permutation classes. All considered groups are finite. In the definitions and notation, we follow [1].

DEFINITION 1[1]. A class \mathfrak{F} is a *Fitting class* if and only if the following two conditions are satisfied:

- 1) if $G \in \mathfrak{F}$ and $N \trianglelefteq G$, then $N \in \mathfrak{F}$;
- 2) if $M, N \trianglelefteq G = MN$ with M and N in \mathfrak{F} , then $G \in \mathfrak{F}$.

Let P be a set of all primes and $\pi \subseteq P$. A π -number is an integer whose prime divisors all belong to π . A subgroup H of a group G is called a *Hall π -subgroup* if $|H|$ is a π -number and $|G:H|$ is a π' -number, where $\pi' = P \setminus \pi$.

A subgroup H of G is called a *Hall subgroup* if it is a Hall π -subgroup for some

$\pi \subseteq P$. Evidently H is a Hall subgroup of G if and only if $(|G:H|, |H|) = 1$.

Let \mathfrak{F} be a Fitting class of a group G . A subgroup V of G is called *\mathfrak{F} -maximal* if the following conditions are satisfied:

- 1) $V \in \mathfrak{F}$;
- 2) if $V \leq U \leq G$ and $U \in \mathfrak{F}$, then $U = V$.

The subgroup V of G is called *\mathfrak{F} -injector* if $V \cap K$ is \mathfrak{F} -maximal subgroup of K for every normal subgroup K of G .

DEFINITION 2 [1]. The operator $L_\pi(\cdot)$. Let π be a set of primes, and let \mathfrak{F} be a Fitting class of finite soluble groups and \mathfrak{S} is a class of all soluble groups. Then define