(3) If $S \in \mathsf{F}$ and $x \in G$, then $S^x \in \mathsf{F}$.

For a Fitting set $\mathsf{F}$ of $G$, the $\mathsf{F}$-injector of $G$ is similarly defined as the $\mathsf{F}$-injector for Fitting class $\mathsf{F}$ (see [1, Definition VIII. (2.5)]).

Hartley [4] proved that for any soluble Fitting class $\mathsf{F}$ (that is, all groups in $\mathsf{F}$ are soluble), every $\mathsf{F}$-injector $V$ of a soluble group $G$ either covers or avoids every chief factor $H/K$ of $G$, that is, either $(V \cap H)K = H$ or $(V \cap H)K = K$.

In this connection, the problem arises in the class of non-soluble groups, describe the cover-avoid property of $\mathsf{F}$-injectors of a group $G$ on its chief factors.

For any set $\mathsf{F}$ of subgroups of $G$, we let $\sigma(\mathsf{F}) = \cup_{G \in \mathsf{F}}\, \sigma(G)$.

It is proved

**Theorem.** Let $\mathsf{F}$ be a Fitting set of a group $G$ and $\varnothing \neq \pi \subseteq \mathbb{P}$. Then every $\mathsf{F}$-injector of $G$ either covers or avoids every chief factor of $G$ in each of the following cases:

1) $G \subseteq \mathsf{F} \circ \mathsf{S}^{\pi}$, where $\pi = \sigma(\mathsf{F})$;
2) $G \subseteq \mathsf{S}^{\pi}$ and $\mathsf{F} \circ \mathsf{E}_{\pi'} = \mathsf{F}$.

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## ON THE POSITIVE INTEGER SOLUTION OF NONLINEAR EQUATIONS $X^2+AX=B$ AND $X^3+AX^2+BX=C$ FOR THE SECOND ORDER MATRICES

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The problem of finding integer positive solution of nonlinear matrix equations of polynomial type for matrices of various orders plays an important role in solving a wide range of problems associated with the modeling of economic, social processes [1, c. 189].

The aim of this work is to find the simplest method for solving matrix equations for the second order matrices.

**Material and methods.** The matrix equation was recorded in the form of a system consisting of four equations, which were solved by analytical methods. In the process of the study, a package of symbolic mathematics Maple 18 was used.

**Results and their discussion.** Consider the nonlinear matrix equation

$$X^2 + AX = B \tag{1}$$

where $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$, $B = \begin{pmatrix} K & L \\ M & N \end{pmatrix}$.

**Lemma 1.** *To find a matrix-root of equation (1), it is necessary for each possible value of the variable a to solve the quadratic equation* $fb^2 + (a + \alpha + g + \beta f)b - L + \beta g = 0$, *for the variable b, where* $f = \frac{m}{k}$, $g = \frac{m\beta - ka - k\delta}{k}$, $k = K - a^2 - \alpha a$, $m = M - \gamma a$; *the variables c and d can be found using the connection formula for these variables with variable d:* $c = \frac{k}{\beta + b}$, $d = fb + g$. *The estimate for the variable a has the form:* $1 \le a \le \sqrt{K - 1 - \alpha - \beta}$.

Proof. In this case, the system of equations corresponding to equation (1) will have the following form:

$$\begin{cases} a^2 + bc + \alpha a + \beta c = K, \\ b(a + d) + \alpha b + \beta d = L, \\ c(a + d) + \gamma a + \delta c = M, \\ bc + d^2 + \gamma b + \delta d = N. \end{cases} \tag{2}$$

We consider the first and third equations of system (2). Introducing the new variables *k* and *m*: $k = K - a^2 - \alpha a$, $m = M - \gamma a$, we obtain a system of equations (3):

$$\begin{cases} bc + \beta c = k, \\ c(a + d) + \delta c = m. \end{cases} \tag{3}$$

Solving the system (3), we can find a formula expressing the relationship between the variables *b* and *d*: $d = fb + g$, where $f = \frac{m}{k}$, $g = \frac{m\beta - ka - k\delta}{k}$. Substituting it into the second equation of system (2), we obtain an equation of the second order with respect to the variable *b*:

$$fb^2 + (a + \alpha + g + \beta f)b - L + \beta g = 0 \tag{4}$$

Solving equation (4) with respect to the variable b, we can find the variable *d*, and the variable *c* can be found using the first equation of system (3).

To find an asymmetric matrix-root of equation (1) under the condition that the matrices A and B are not symmetric matrices, it is necessary for each possible value of the variable *a* to solve the quadratic equation for the variable *b* and using the coupling formulas to find the values of the variables *c* and *d*.

Consider the nonlinear matrix equation

$$X^3 + AX^2 + BX = C \tag{5}$$

where $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$, $B = \begin{pmatrix} \varepsilon & \varphi \\ \psi & \mu \end{pmatrix}$, $C = \begin{pmatrix} K & L \\ M & N \end{pmatrix}$.

**Lemma 2.** *To find a matrix-root of equation (5), it is necessary for the variable b to solve the quadratic equation* $psb^2 + (kr - np + sq)b - nq = 0$, *where* $p = 2a + d + \alpha$, $r = a + 2d + \delta$, $q = \beta(a + d) + \varphi$, $s = \gamma(a + d) + \psi$, $k = K - a^3 - \alpha a^2 - \varepsilon a$, $n = N - d^3 - \delta d^2 - \mu d$. *The variable c can be found using the relation formula with the variable b:* $c = \dfrac{k}{bp + q}$. *The estimate for the variable a has the form:* $1 \le a \le \sqrt[3]{K - 3 - 2\alpha - \varepsilon - 2\beta - \varphi}$. *The estimate for the variable d has the form:* $1 \le d \le \sqrt[3]{N - 3 - 2\gamma - \psi - 2\delta - \mu}$.

Proof. In this case, the system of equations corresponding to equation (5) will have the following form:

$$\begin{cases} a^3 + 2abc + bcd + \alpha a^2 + \alpha bc + \beta ac + \beta cd + \varepsilon a + \varphi c = K, \\ a^2 b + b^2 c + abd + bd^2 + \alpha ab + \alpha bd + \beta bc + \beta d^2 + \varepsilon b + \varphi d = L, \\ a^2 c + bc^2 + acd + cd^2 + \gamma a^2 + \gamma bc + \delta ac + \delta cd + \psi a + \mu c = M, \\ abc + 2bcd + d^3 + \gamma ab + \gamma bd + \delta bc + \delta d^2 + \psi b + \mu d = N. \end{cases} \tag{6}$$

Consider the first and fourth equations of system (6):

$$\begin{cases} bcp + cq = k, \\ bcr + bs = n. \end{cases} \tag{7}$$

where $k = K - a^3 - \alpha a^2 - \varepsilon a$, $n = N - d^3 - \delta d^2 - \mu d$, $p = 2a + d + \alpha$, $q = \beta(a + d) + \varphi$, $r = a + 2d + \delta$, $s = \gamma(a + d) + \psi$.

Expressing the variable *c* from the first equation of system (7), we obtain:

$$c = \frac{k}{bp + q} \tag{8}$$

Substituting expression (8) into the second equation of system (7), we obtain an equation of the second order with respect to the variable *b*:

$$psb^2 + (kr - np + sq)b - nq = 0 \tag{9}$$

To solve the problem it is necessary to solve the quadratic equation (9) for each possible pair of values of the variables *a* and *d* to find the variable *b*, using the formula (8), to find the value of the variable *c*.

To solve the matrix equation (5), we can use the following program written in Pascal.

```
Var W, X, Y, Z, a, d, m1, t1, alpha, beta, gamma, delta, epsilon, fi, psi, mu, k, n: integer;  b1, b2, c1, c2, m, t, p, q, r, s, dis, u1, u2, v1, v2: real;
begin
read( W, X, Y, Z, alpha, beta, gamma, delta, epsilon, fi, psi, mu);
m:=power(W-3-2*alpha-epsilon-2*beta-fi, 1/3); m1:=trunc(m);
t:=power(Z-3-2*gamma-psi-mu-2*delta, 1/3); t1:=trunc(t);
for a:=1 to m1 do begin   k:=W-a*a*a-alpha*a*a-epsilon*a;
for d:=1 to t1 do begin     n:=Z-d*d*d-delta*d*d-mu*d;
p:=2*a+d+alpha;     q:=beta*(a+d)+fi;     r:=a+2*d+delta; s:=gamma*(a+d)+psi;
dis:=(k*r-n*p+s*q)*(k*r-n*p+s*q)+4*p*s*n*q;
if dis>0 then begin
b1:=(-k*r+n*p-s*q+sqrt(dis))/(2*p*s); c1:=k/(b1*p+q);
b2:=(-k*r+n*p-s*q-sqrt(dis))/(2*p*s); c2:=k/(b2*p+q);
u1:=frac(b1); u2:=frac(c1); v1:=frac(b2); v2:=frac(c2); end;
if (b1>0) and (c1>0) and (u1=0) and (u2=0) then begin
writeln(a); writeln(b1); writeln(c1); writeln(d); writeln('Next') end;
if (b2>0) and (c2>0) and (v1=0) and (v2=0) then begin
writeln(a); writeln(b2); writeln(c2); writeln(d); writeln('Next'); end;
end; d:=d+1; end; a:=a+1; end.
```

**Conclusion.** It was shown that we can use analytical methods for solving the problem of finding positive integer solution of matrix equations of polynomial type for second-order matrices.

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