To control the robot arm, namely stepper motors, ROS will be installed on the Ubuntu operating system. ROS (Robot Operating System) is a robot programming framework that provides functionality for the distributed operation of all programmable robot systems. ROS is based on graph architecture, where data processing occurs at nodes that can receive and transmit messages between themselves.

The advantages of ROS allow processing and connecting complex sensors to the robot, which speeds up and simplifies the development of software modules for the end device.

Conclusion. The analysis of the modern market of robotic mechanisms for various spheres of human activity showed that it is necessary to use advanced software and current technologies that will allow to adapt manipulators for use in everyday life. Creating a low-cost robotic manipulator is possible by using the Raspberry Pi board in conjunction with the Arduino to control the robot itself and when printing robot parts by printing 3D models on a 3D printer.

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ON THE APPLICATION OF THE TCHIRNHAUS TRANSFORMATIONS FOR AN ALGEBRAIC EQUATION OF THE THIRD DEGREE

Mikhail Chernyavsky

VSU named after P.M. Masherov, Vitebsk, Belarus

From the history of mathematics, it is known that E. V. von Tchirnhaus in 1683 published his own method of solving algebraic equations of various degrees. His goal was to obtain an algorithm for solutions an algebraic equation of the fifth degree [1, p. 166]. Nevertheless, Tchirnhaus's ideas and results can be used to more easily solve algebraic equations of the third and fourth degrees. However, in most authoritative literature on the theory of polynomials and the solution of algebraic equations, only general ideas of the Tchirnhaus transformations are given and there are no specific analytical intermediate formulas. This fact significantly reduces the practical value of the investigated method.

So, the aim of the research is to obtain in a convenient form all the intermediate formulas involved in the Tchirnhaus's transformations for a cubic algebraic equation.

Material and methods. The material of the research is cubic cubic algebraic equations and Tchirnhaus's transformations for polynomials. Research methods – methods of algebra and mathematical analysis.

Findings and their discussion. Let

$$x^3 + a_1x^2 + a_2x + a_3 = 0 \ \_ \qquad (1)$$

be a cubic equation without multiple roots; $x_j \ (j=1,2,3)$ – the roots of this equation. We get a new equation

$$y^3 + b_1y^2 + b_2y + b_3 = 0 \qquad (2)$$

whose roots are numbers

$$y_j = p_0x_j^2 + p_1x_j + p_2 \quad (j=1,2,3). \qquad (3)$$

Such an equation can be written as

$$\left(y - p_0x_1^2 - p_1x_1 - p_2\right)\left(y - p_0x_2^2 - p_1x_2 - p_2\right)\left(y - p_0x_3^2 - p_1x_3 - p_2\right) = 0. \qquad (4)$$

The difficulty is to express the coefficients of equation (4) explicitly in terms of $a_j\,(j=1,2,3)$, $p_k\,(k=0,1,2)$.

Theorem. The following equalities hold:

$$b_1 = \left(2a_2 - a_1^2\right)p_0 + a_1p_1 - 3p_2; \qquad (5)$$

$$b_2 = \left(a_2^2 - 2a_1a_3\right)p_0^2 + \left(3a_3 - a_1a_2\right)p_1 + 2\left(a_1^2 - 2a_2\right)p_0p_2 + a_2p_1^2 - 2a_1p_1p_2 + 3p_2^2; \qquad (6)$$

$$b_3 = -a_3^2p_0^3 + a_3p_1^3 + a_2a_3p_0^2p_1 + (2a_1a_3 - a_2^2)p_0^2p_2 - a_1a_3p_0p_1^2 + \left(a_1a_2 - 3a_3\right)p_0p_1p_2 -$$
$$-\left(a_1^2 - 2a_2\right)p_0p_2^2 - a_2p_1^2p_2 + a_1p_1p_2^2 - p_2^3. \qquad (7)$$

Proof. We express the coefficients of equation (4) in terms of $x_j\ (j=1,2,3)$, $p_k\ (k=0,1,2)$:

$$b_1 = -\left(x_1^2 + x_2^2 + x_3^2\right)p_0 - \left(x_1 + x_2 + x_3\right)p_1 - 3p_2;$$

$$b_2 = \left(x_1^2x_2^2 + x_1^2x_3^2 + x_2^2x_3^2\right)p_0^2 + \left(x_1x_2^2 + x_1x_3^2 + x_1^2x_2 + x_1^2x_3 + x_2x_3^2 + x_2^2x_3\right)p_1 +$$
$$+2\left(x_1^2 + x_2^2 + x_3^2\right)p_0p_2 + \left(x_1x_2 + x_1x_3 + x_2x_3\right)p_1^2 + 2\left(x_1 + x_2 + x_3\right)p_1p_2 + 3p_2^2;$$

$$b_3 = -x_1^2x_2^2x_3^2p_0^3 - x_1x_2x_3\left(x_1x_2 + x_1x_3 + x_2x_3\right)p_0^2p_1 - (x_1^2x_2^2 + x_1^2x_3^2 + x_2^2x_3^2)p_0^3p_2 -$$
$$-x_1x_2x_3(x_1 + x_2 + x_3)p_0p_1^2 - (x_1^2x_2 + x_1^2x_3 + x_1x_2^2 + x_1x_3^2 + x_2^2x_3 + x_2x_3^2)p_0p_1p_2 -$$
$$-(x_1^2 + x_2^2 + x_3^2)p_0p_2^2 - x_1x_2x_3p_1^2 - (x_1x_2 + x_1x_3 + x_2x_3)p_1^2p_2 - (x_1 + x_2 + x_3)p_1p_2^2 - p_2^3.$$

Equality (5) follows directly from Vieta's formulas. To prove equality (6), we must express the coefficients in b_2 for p_0^2 and p_1 in terms of $a_j\ (j=1,2,3)$. The coefficient that stands for p_0^2 has the form $x_1^2x_2^2 + x_1^2x_3^2 + x_2^2x_3^2$. Applying the theorem on the representation of the symmetric polynomial of many variables in terms of elementary symmetric polynomials [2, p. 322], we obtain equality

$$x_1^2x_2^2 + x_1^2x_3^2 + x_2^2x_3^2 = a_2^2 - 2a_1a_3.$$

Similarly, we obtain the equality for the coefficient for p_1:

$$x_1x_2^2 + x_1x_3^2 + x_1^2x_2 + x_1^2x_3 + x_2x_3^2 + x_2^2x_3 = 3a_3 - a_1a_2.$$

The coefficients for p_0p_2, p_1^2, p_1p_2 are obviously expressed from the Vieta's formulas.

Similarly, we get equality (7). The theorem is proven.

Next, it is natural to equate the coefficients b_1 and b_2 to zero, which allows us to go from equation (2) to an equation of the form

$$y^3 + b_3 = 0.$$

To simplify the calculations, we take the coefficient $p_0 = 1$ in all the above formulas. $p_0 = 1$ Also, without limiting the commonality in the original equation (1), you can take the $a_1 = 0$ factor, because in practice you can always do it with a known substitution $x = z - a_1/3$.

Given these simplifications, from equation $b_1 = 0$ we unequivocally find

$$p_2 = \frac{2}{3}a_2.$$

Substituting p_2 into equation $b_2 = 0$, it is easy to obtain that

$$p_1 = \frac{-9a_3 \pm \sqrt{12a_2^3 + 81a_3^2}}{6a_2}.$$

Consider a particular numerical example.

$$x^3 + 9x - 16 = 0.$$

For him we calculate

$$p_1 = \frac{8}{3} + \frac{\sqrt{91}}{3} \approx 5{,}846464;\quad b_3 = -\frac{66248}{27} - \frac{5824\sqrt{91}}{27} \approx -4511{,}311818.$$

We calculate three values of cubic root from b_3:

$$y_1 \approx 16{,}523458;\quad y_2 \approx -8{,}261729 + 14{,}309734i;\quad y_3 \approx -8{,}261729 - 14{,}309734i.$$

Substituting successively obtained values of y into equation (3), we find from it in the general case 6 different values of x. From these values, we choose those that turn the original equation (1) into identity. In the above example, this

$$x_1 \approx 1{,}443545;\quad x_{2,3} \approx -0{,}7217726 \pm 3{,}250056i.$$

Conclusion. Thus, in the course of the work, all intermediate formulas involved in the Tchirnhaus's transformations for a cubic algebraic equation are obtained in an analytical form.

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