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О некоторых взаимосвязях между вирусологией и математикой

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Работа посвящена приложению некоторых аспектов теории чисел к описанию групп вирусов посредством геометрических симметрий. Для этой цели используются формулы для решения в нечетных числах диофантова уравнения Люнгрена. Доказано, что все решения уравнения $x^2+3y^2=4\ R^2+35^2$, $R,S\ =1$ *находятся с помощью формул* $x=|R-3S|,\quad y=R+S$ *или* $x=|R+3S|,\quad y=|R-S|$. *Полученный результат применен к описанию серии групп вирусов посредством симметронов.*

Ключевые слова: *Зеленогурский университет, симметрон, уравнение, нечетные числа.*

On some connections between virology and mathematics

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The paper is devoted to the application of some aspects of the theory of numbers to the description of groups of viruses by means of geometric symmetries. With this purpose formulas for solution in odd numbers of the Diophantine equation of Lungrene are used. It is proved that all the solutions of the equation $x^2+3y^2=4\ R^2+35^2$, $R,S\ =1$ *are found with the help of the formulas* $x=|R-3S|,\quad y=R+S$ *or* $x=|R+3S|,\quad y=|R-S|$. *The obtained result is applied to the description of the series of virus groups by means of symmetrones.*

Key words: *University of Zielona Góra, symmetrone, equation, odd numbers*.

1. Introduction. In virology are known diffe-rent groups of viruses. One of such groups has been found by Stoltz [1–2] and by Wrigley [3–4] and is called as **symmetrons.** Virus particles are invariably enclosed by shells of protein subunits and these are packed geometrically according to symmetry rules. Goldberg in the paper [5] indicated that total number of nearly identical subunits which may be regularly packed on the closed icosahedral surface is given by the following formula:

$$\mathbf{N = 10T + 2 = 10(a^2+ab+b^2) + 2}$$

(Goldberg's formula), (1.1)

where **a, b** are given non-negative integer numbers and

$$\mathbf{T = a^2+ab+b^2} \qquad (1.2)$$

is called as **triangular number** for corresponding symmetron.

Stoltz and Wrigley discovered that the symmetrons have the construction of linear, triangular and pentagonal and are called; respectively:

disymmetrons;	$\mathbf{d_u = u-1,}$	$\mathbf{d_u = u-1,2,3,..}$	
trisymmetrons;	$\mathbf{t_v = (v-1)v/2,}$	$\mathbf{v=1,2,3,..}$	**(S–W)**
pentasymmetrons;	$\mathbf{p_w=5(w-1)w/2+1,..}$	$\mathbf{w=1,2,3,..}$	

It is known that an icosahedron has:

30 axes of twofold symmetry,

20 of threefold symmetry and

12 of fivefold symmetry.

Therefore the subunits on the surface of an icosahedral virus may be divided into **30,20,12** previously listed groups symmetry. Suppose that

the **30** disymmetrons contain $\mathbf{d_u}$ subunits,

the **20** trisymmetrons contain $\mathbf{t_v}$ subunits and

the **12** pentasymmetrons contain $\mathbf{p_w}$ subunits.

The we obtain

$$\mathbf{N = 10T + 2 = 30d_u + 20t_v + 12p_w} \qquad (1.3),$$

where $\mathbf{d_u}$ **,** $\mathbf{t_v}$ **,** $\mathbf{p_w}$ are given by the formulas **(S–W).**

We note that for each value of **N** given by the formula **(1.1)** the number **f(N)** of the solutions of the equation **(1.3)** is the number theoretically possible ways of making a virus with N subunits but with different combinations of symmetrons. Putting

$$\mathbf{x = 2v–1, \quad y = 2w–1, \;\; z = u–1}$$

and using the formulas **(S–W)** we can transformed the equation **(1.3)** to the following form:

$$\mathbf{x^2 + 3y^2 + 12z = 4T = 4(a^2+ab+b^2).} \qquad (1.4)$$

2. Ljunggren's problem. In 1974 in the paper [6] which has been presented by N.G. Wrigley and V. Brun, Ljunggren posed the following problem:

Find all odd positive integers x, y and non-negative integers z satisfying the equation (1.4) for given non-negative integers values of a and b. (L–P)

It is easy to see that the equation **(1.4)** is equivalent the following equation;

$$\mathbf{x^2+3y^2 = 4T – 12z = 4(T – 3z);} \quad (2.1)$$

where $\mathbf{T=a^2+ab+b^2}$ is given triangular number.

Since $\mathbf{x^2+3y^2 >0}$ **and z** is non-negative integer number then by (2.1) it follows that

$\mathbf{T–3z > 0}$ and consequently there is only finite number of non-negative numbers **z** belonging to the interval **[0,T/3).** Let $\mathbf{z = z_0}$ be such one the element of this interval.

Then we have $\mathbf{T_0 = T – 3z_0}$, and the equation **(2.1)** has the following form:

$$\mathbf{x^2+3y^2 =4T_0 .} \qquad (2.2)$$

Since $\mathbf{T_0}$ is positive integers then using many property from number theory and Diophantine equation we transform the equation (2.2) to the following form:

$$\mathbf{x^2 + 3y^2 = 4(R^2 + 3S^2) \; ; (R,S)=1.} \qquad (2.3)$$

The procedure concerning this transform is given in our paper [7]. In this paper has been presented computational method for determination **x, y** when we determine **R,S.** Namely we use of the following inequality:

$$\mathbf{x < 4max\{R,S\} , y < 3max\{R,S\} .}$$

3. Solution of the Ljunggren problem. In the paper [8] has been proved the following Theorem:

Theorem. *All solutions in odd positive integers **x, y** of the equation **(2.3)** are given by the following formulas:*

$$x=|R–3S|, y = R+S \;\; or \;\; x=R+3S, y=|R–S| \qquad (3.1).$$

This **Theorem** give full solution of the Ljunggren problem.

Remark. For application of this result we note that the number $\mathbf{R^2+3S^2}$ must be odd integer number. Indeed, since **x, y** must be odd integers then it is well-known that $\mathbf{x^2}$ give the residue equal to **1 (mod 8)** and $\mathbf{y^2}$ also give the residue equal to **1(mod8).** Therefore $\mathbf{x^2 + 3y^2}$ give the residue equal **4(mod8).** But right hand of the equation give the residue **0(mod8),** because if $\mathbf{R^2+3S^2=2k}$, then we have **4(R2+3S2)=4.2k=8k** and **8k** give the residue **0(mod8).**

Example. Consider the symmetron – **Reovirus,** which has the triangular number **T=13,**

with **a=3** and **b=1**, so $\mathbf{T=a^2+ab+b^2 =3^2 +3.1+1^2 =9+3+1=13.}$

The equation **(2.1), (2.3)** has the form:

$$\mathbf{x^2+3y^2= 4(13–3z) =4(R^2+3S^2),} \qquad \mathbf{z < 13/3;}$$

From the remark follows that **13–3z > 0** must be odd integer number.

Hence we have;

z = 0,2,4.

Let **z=0.**

Then we have

$$\mathbf{x^2+3y^2=4.13 = 4.(1^2+3.2^2),} \qquad (3.2)$$

hence **R=1,S=2.** By the formula **(3.1)** it follows that

x=|R–3S|=|1–3.2|=5, y=R+S=1+2=3

or x=R+3S=1+3.2=7, y=|R–S|=1.

Consequently we obtain two solutions **(x, y)=(5,3);(7,1)**

and easy to see that those solutions satisfy the equation **(3.2).**

Let **z=2.**

Then we have $\mathbf{13–3z=13–6=7=2^2 +3.1^2}$ and consequently we have **R=2,S=1.**

From the formula **(3.1)** we have

x=|R–3S|=1, y=R+S=3

or x=R+3S=2+3=5,y=|R–S|=|2–1|=1.

Let **z=4.**

Then we have

$$\mathbf{13–3z=13–3.4=13–12=1\#R^2+3S^2},$$

because **(R,S)=1.**

Now we can determined the number disymmetrons, trisymmetrons and pentasymmetrons if **z=0.** For the solution **x=5,y=3** we obtain from the formulas **x=2v–1,y=2w–1,z=u–1** that

v=3,w=2,u=1. By the formulas **(S–W)** we obtain that

$$\mathbf{t_v=3,p_w=6,d_u=0.}$$

Therefore we have

$\mathbf{N=10T+2=10.13+2=132=30d_u+20t_v+12p_w=30.0+20.3+12.6=60+72=132.}$

For this case we give in the second part of this paper corresponding graphic this virus.

In the second solution **x=7,y=1**,when **z=0** we obtain that **v=4,w=1,u=1** and **t_v=6,p_w=1,d_u=0.**

Hence, **N=30.d_u+20.t_v+12.p_w=30.0+20.6+12.1= =120+12=132.**

If **z=2** then we have **x=1,y=3** and we get **v=1,w=2,u=3.** From the formulas **(S–W)** follows that **t_v=0,p_w=6,d_u=2** and consequently we have **N=30.2+20.0+12.6=60+72=132.**

In similar way we can determined the structure this symmetron for the second case of the solution when **x=5,y=1**, when **z=2.** In this case we obtain **v=3,w=1,u=3** and consequently we get that **d_u=2,t_v=3,p_w=1.** Hence,

N=30.2+20.3+12.1=60+60+12=132.

In the second part of this paper etitled: «Examples of some type of symmetrons» we present graphic structure some viruses with determined the number of disymmetrons, trisymmetrons and pentasymmetrons.

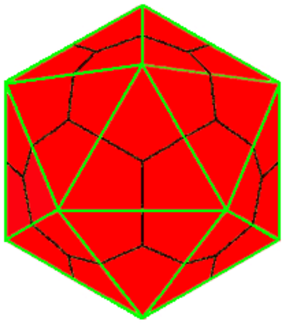
Examples of some type of symmetrons

1. Parvovirus – T1

We have T = 1, a = 1, b = 0, hence

N = 10T + 2 = 10.1 + 2 = 12

and 12 = 30.0 + 20.0 + 12.1

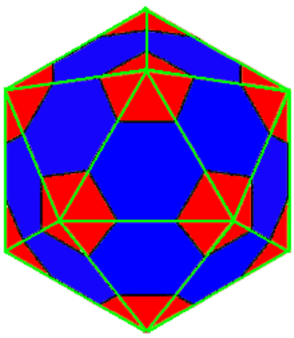


Pentasymetrons have the colour red

2. Poliovirus – T3

We have T = 3, a = 1, b = 1, hence

N = 10T + 2 = 10.3 + 2 = 30.0 + 20.1 + 12.1 = 32

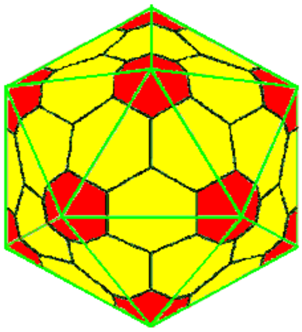


Pentasymetrons have the colour red, trisymetrons have the colour blue

3. Togavirus – T4

We have T = 4, a = 2, b = 0, hence

N = 10.4 + 2 = 42 = 30.1 + 20.0 + 12.1

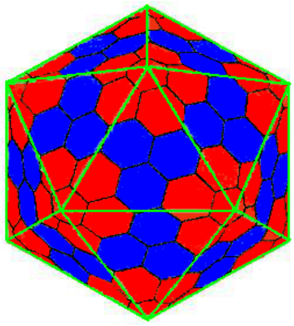


Pentasymetrons have the colour red, disymetrons have the colour yellow

4. Reovirus – T13

We have T = 13, a = 3, b = 1, hence

N = 10.13 + 2 = 132 = 30.0 + 20.3 + 12.6

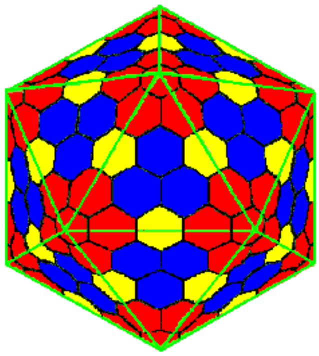


Pentasymetrons have the colour red, trisymetrons have the colour blue

5. Herpesvirus – T16

We have T = 16, a = 4, b = 0, hence

N = 10.16 + 2 = 162 = 30.1 + 20.3 + 12.6

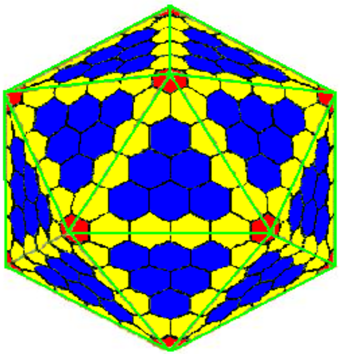


Pentasymetrons have the colour red, trisymetrons have the colour blue and disymetrons have the colour yellow

6. Adenovirus – T25

We have $T = 25$, $a = 5$, $b = 0$, ***hence***

$$N = 10.25 + 2 = 252 = 30.4 + 20.6 + 12.1$$



Pentasymetrons have the colour red, trisymetrons have the colour blue, disymetrons have the colour yellow

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